

The $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ mesons in $\tilde{U}(12)$ -classification scheme of hadrons

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The narrow mesons, $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$, observed recently in the final states $D_s^+ \pi^0$ and $D_s^{*+} \pi^0$ are pointed out to be naturally assigned as the ground-state scalar and axial-vector chiral states in the $(c\bar{s})$ system, which would newly appear in the covariant $\tilde{U}(12)$ hadron-classification scheme proposed a few years ago. We predict the comparatively large electromagnetic decay widths to other models, which are due to the intrinsic electric dipole moment. The SELEX state $D_{sJ}(2632)$ is also able to be assigned to the P -wave chiral state with $J^P = 1^-$ in the $\tilde{U}(12)$ -classification scheme.

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I. INTRODUCTION

A. Present status of hadron spectroscopy and $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ mesons

There exist the two contrasting, non-relativistic and relativistic approaches for describing composite hadrons: The former is based on the non-relativistic quark model(NRQM) with the approximate $SU(6)_{SF} \times O(3)_L$ symmetry(S , F and L denoting Pauli-spin, flavor and orbital angular-momentum of constituent light-quarks, respectively) and gives a theoretical base to the particle data group(PDG) level-classification[1], while the latter is based on the field theory with the spontaneously-broken chiral symmetry. It is widely accepted that π meson octet has the property as a Nambu-Goldstone boson in the case of spontaneous breaking of chiral symmetry.

Owing to the recent progress, both theoretical and experimental, the existence of light σ -meson as chiral partner of π -meson seems to be established especially through the analysis of various $\pi\pi$ -production processes[2]. This gives a strong support for the relativistic approach and

suggests to assign a series of low-mass scalar mesons as the scalar σ -nonet $\{a_0(980), \sigma(600), f_0(980), \kappa(900)\}$ in the $q\bar{q}$ ground states.

The difficulty here of the conventional $SU(6)_{SF} \times O(3)_L$ scheme in the framework of NRQM is that there are no appropriate seats for the σ nonet in the $(q\bar{q})$ ground state. More seriously, it is, in principle, not able to treat the chiral symmetry, and it could not refer to the well-known property of π and σ meson nonets as being mutual chiral partners. In order to treat the chiral symmetry, the manifestly Lorentz-covariant character of the relevant scheme is indispensable, since the generator of chiral transformation for constituent quarks is defined as $\gamma_5 \equiv \gamma_1 \gamma_2 \gamma_3 \gamma_4$.

Recent discovery of new narrow resonant states $D_{sJ}^*(2317)/D_{sJ}(2460)$ [3] causes further a serious problem in hadron physics: They have the quantum numbers, $J^P = 0^+/1^+$, respectively. However, their mass values are too low to be assigned as the corresponding $(c\bar{s})$ P -wave states in NRQM, although moderate as the S -wave states(, where missing positive-parity seats). Accordingly, they are mostly interpreted as $4q$ -states[4, 5], DK/D^*K [6] and $D_s\pi$ [7] molecular states, or singularities of unitarized meson-meson scattering amplitudes in chiral models[8].

In the preceding works[9, 10], the authors presupposed

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the existence of *degenerate* two heavy-spin multiplets, forming a linear representation of chiral symmetry and consisting of $J = 0^-$ and 1^- members, $H(0^-, 1^-)$ and $H'(0^+, 1^+)$, where H particles are assigned to the conventional ground state P_s and V_μ mesons, and H' particles are to the relevant scalar S and axial-vector A_μ mesons, respectively. Then the authors[9] derived the interesting results on the mass-splitting between the chiral partners H and H' . However, the identification of these particles $H'(0^+, 1^+)$ in the level-classification scheme is obscure, although the authors, in application to radiative decay processes, assigned the H' to the P-wave $j_q^P = \frac{1}{2}^+$ multiplet in the heavy quark effective theory(HQET).

A few years ago we have proposed a manifestly covariant scheme, the $\tilde{U}(12)$ -classification scheme[11], which maintain on the one hand the successful part of $SU(6)_{SF} \times O(3)_L$ scheme and, on the other hand, it reconciles the quark model to the chiral symmetry. It has a unitary symmetry in the hadron rest frame, “static $U(12)$ ”, embedded in the covariant $\tilde{U}(12)$ -tensor space(see, the next-subsection).

The static $U(12)$ includes both its subgroup, $SU(6)_{SF}$ and chiral $SU(3)_L \times SU(3)_R$, as $U(12) \supset SU(6)_{SF} \times SU(2)_\rho$ on the one hand, and as $U(12) \supset SU(3)_L \times SU(3)_R \times SU(2)_\sigma$ on the other hand. The $SU(2)_\rho$ and $SU(2)_\sigma$ are Pauli’s spin groups concerning the boosting and the intrinsic-spin rotation, respectively, of the constituent quarks, being connected with decomposition of Dirac γ -matrices, $\gamma \equiv \rho \otimes \sigma$. The freedom on $SU(2)_\rho$, which is indispensable for covariant description of spin one-half particles, plays also an important role to define the rule of chiral transformation for general quark-composite hadron. This becomes possible by introduction of chiral spinors into the complete set of fundamental Dirac-spinors of the $\tilde{U}(12)$ tensor space, as will be explained in the following sub-section.

The purpose of this work is to investigate the properties of the controversial $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ mesons in the framework of our new scheme. The heavy-light-quark meson system is the most suitable for testing the validity of our scheme, since it is expected to show clearly both the non-relativistic and relativistic behaviors concerning the heavy and light constituent quarks, that is, the heavy quark symmetry(HQS) and the light quark chiral symmetry, respectively.

In our scheme, the degenerate, two heavy-spin multiplets $H(0^-, 1^-)$ and $H'(0^+, 1^+)$ above mentioned are assigned to be the ground states ($c\bar{s}$) mesons, where H and H' , being the eigen-states of ρ_3 -spin(concerning the light constituent quarks) with the eigen-values, $r = +$ and $-$, respectively, become mutually chiral partners, while their members belong to the multiplets with $S = (0, 1)$ of $SU(2)_\sigma$.

Thus, in our covariant classification scheme, the relevant $D_{sJ}^*(2317)/D_{sJ}(2460)$ are *naturally assigned as the scalar/axial-vector chiralons, playing the role of chiral partners of pseudo-scalar/vector Paulons, $D_s(1968)/D_s^*(2112)$, in a linear representation of chi-*

ral symmetry for light quarks. All these particles are assigned as the S-wave ($c\bar{s}$) ground states, and the ordinary P-wave $j_q^P = \frac{1}{2}^+$ multiplet with $J^P = (0^+, 1^+)$ is expected to exist as different particles in the slightly-higher mass region. (see, the discussion below Eq.(15).)

Furthermore, it will be shown that their properties predicted from our scheme are almost consistent with the present experiments.

B. Lorentz-covariance and overlooked freedom $SU(2)_\rho$ of composite hadrons

In this sub-section we recapitulate fundamental ideas and basic-points of our scheme, which is described rigorously in the sections II and III. In our scheme we start from the master Klein-Gordon(K.G.) equation on the wave function(WF) of composite hadrons with the *squared-mass* $\mathcal{M}^2(r_\mu)$ operator, acting on the Lorentz-space $O(3, 1)_L$ for relative space-time coordinates r_μ ’s of constituents. Here we should like to mention that the observable entity is not the *confined quarks but the composite hadrons*.

Aside from the center of mass(CM) plane-wave motion, the internal WF of hadrons are given, in the ideal limit, as eigen-functions of the $\mathcal{M}^2(r_\mu)$, which is taken to be the covariant oscillator of Yukawa-type[12]. Their spectra simulates, by imposing a subsidiary condition[13] to freeze the relative-time freedom, those of oscillator on $O(3)_L$ in NRQM, and the WF represents the Lorentz-contraction effects[14] due to CM motion.

Further, the WF of hadrons are set up(for a while considering light-quarks) to be appropriate tensors in $\tilde{U}(12) \times O(3, 1)_L$ -space, reflecting the quark-composite structure of relevant hadrons. Here $\tilde{U}(12)$ includes the subgroups $\tilde{U}(12) \supset \tilde{U}(4)_{DS} \times SU(3)_F(\tilde{U}(4)_{DS})$ being the pseudo-unitary homogeneous Lorentz-group \mathcal{L}_4 for the Dirac spinors).

Then the spin-flavor WF of relevant hadrons become covariant tensors in the $\tilde{U}(12)$ -space with definite four-velocity $v_\mu \equiv P_\mu/M(P_\mu(M))$ being the four momentum(mass) of the hadrons)

The spin WF of hadrons are tensors in $\tilde{U}(4)_{DS}$ space, and represented by relevant multi-product of the original Dirac spinors (as fundamental vectors in $\tilde{U}(4)_{DS}$ -tensor space), $W_\alpha^{(\pm)}(v)$ and its Pauli-conjugates $\bar{W}^{\beta(\pm)}(v)$. Here, $W_\alpha^{(+)}(v)$ and $W_\alpha^{(-)}(v)$ denote positive and negative frequency parts, respectively, of the local Klein-Gordon equations for spin one-half particles corresponding to constituent-quarks.:

$$\begin{aligned} W_\alpha(X) &\equiv \sum_{P_\mu(P_0>0)} (e^{iP \cdot X} W_\alpha^{(+)}(P) + e^{-iP \cdot X} W_\alpha^{(-)}(P)), \\ W_\alpha^{(+)}(P) &= \{u_{r=+,s,\alpha}(P), v_{\bar{r}=-,\bar{s},\alpha}(P); s, \bar{s} = (+, -)\}, \\ W_\alpha^{(-)}(P) &= \{v_{\bar{r}=+,\bar{s},\alpha}(P), u_{r=-,s,\alpha}(P), ; \bar{s}, s = (+, -)\}. \\ &-(\gamma_\mu \partial_\mu + M)W(X) = 0. \end{aligned} \quad (1)$$

Both $W_\alpha^{(+)}(P)$ and $W_\alpha^{(-)}(P)$ consist, respectively, of four members as above, where $r(\bar{r}) = \pm$ and $s(\bar{s}) = \pm$ denote the eigen-values of $\rho_3(\bar{\rho}_3)$ and $\sigma_3(\bar{\sigma}_3)$ in the rest frame of hadrons ($\mathbf{v} = \mathbf{0}$) (see Eqs.(17) through (20)).

The positive- and negative-frequency parts of the hadron WF $\Phi(X, r)$ (its Pauri-conjugate $\bar{\Phi}$) are second-quantized as annihilation(creation) operator and creation(annihilation) operators of relevant hadrons (its anti-particles), in conformity with the conventional crossing rule for hadrons.

The meaning of the static $U(12)$, embedded in the covariant $\tilde{U}(12)$ -tensor space (here, $U(4)_{DS}$ embedded in the $\tilde{U}(4)_{DS}$), mentioned in the last sub-section, is that the $\tilde{U}(4)_{DS}$ -invariant squared-mass term in action integral become static $U(4)$ -symmetric by inserting a Lorentz-invariant factor, the unitarizer $F_U(X)$, between the trace of light-quark spin-indices (see Eq.(24)).

Here it is to be remarked on the important fact which has been thus far overlooked: conventionally the spinors $u_{-,s}(v_{-,s})$ for quarks(anti-quarks) are identified with the spinor $v_{+,s}(u_{+,s})$ for anti-quarks(quarks), being based on the hole-theory in the case of free quarks. However, it is not applicable to the confined quarks, coexisting with the other quarks (see II C), and all the above four-members, separately, of $W_\alpha^{(+)}(v)$ for quarks and of $W_\alpha^{(-)}(v)$ for anti-quarks are required as the members of complete set of fundamental vectors to describe the spin WF of hadrons. This is one of the corner-stones of our scheme, and comes from our application of Klein-Gordon equation as the master equation, describing the WF of *observable hadrons*.

This fact is shown as follows: we start from the K.G. equation for Dirac spinor $\psi(x)$ with four-components as

$$(\square - \kappa^2)\psi(x) = (\gamma_\mu \partial_\mu + \kappa)(\gamma_\mu \partial_\mu - \kappa)\psi(x) = 0. \quad (2)$$

This has the two types of solution, which satisfy, respectively, the following Dirac-type equations as

$$(\gamma_\mu \partial_\mu + \kappa)\psi_+(x) = 0, (\gamma_\mu \partial_\mu - \kappa)\psi_-(x) = 0. \quad (3)$$

As is clearly seen from these equations, the ψ_+ and ψ_- form a chiral doublet, whose spinor wave functions, $\psi_{+,\alpha}(v)$ and $\psi_{-,\alpha}(v)$, consist of totally the eight ($8 = 4 \times 2$) independent-ones, and are evidently equivalent to $\{u_{r=+,s,\alpha}(v), v_{\bar{r}=+,\bar{s},\alpha}(v)\}$ and $\{u_{r=-,s,\alpha}(v), v_{\bar{r}=-,\bar{s},\alpha}(v)\}$, respectively.

Here, it may be worthwhile to note that the above reasoning led to the chiral doublet, ψ_+ and ψ_- , is similar to that in the case of deriving the chiral-invariant weak-currents of V-A type[15].

The freedom on $SU(2)_\rho$, concerning discrimination between the eigen-states of ρ_3 -spin of fundamental vectors $W_\alpha^{(\pm)}$'s of $\tilde{U}(4)_{DS}$ -tensor space (which might be vanished in the case of not-confined constituent-quarks) plays important roles in composite hadrons as follows:

(i) In defining the booster for special Lorentz-

transformation, the $SU(2)_\rho$ space is indispensable as

$$S_B(\mathbf{P}) = e^{-i\mathbf{b}\cdot\mathbf{K}}, K_i = \frac{i}{2}\sigma_i \otimes \rho_1. \\ (\mathbf{b} \equiv \hat{\mathbf{v}} \cosh^{-1} v_0). \quad (4)$$

(ii) The transformation for reflection of space-time coordinates is given, as

$$X_\mu \rightarrow X'_\mu = -X_\mu; \\ W_\alpha(X) \rightarrow W'_\alpha(X') = W'_\alpha(-X) = (-\gamma_5 W(-X))_\alpha, \\ -\gamma_5 W_{\pm,s}(v_\mu) = \rho_1 W_{\pm,s}(v_\mu) = W_{\mp,s}(v_\mu). \quad (5)$$

(iii) The transformation for reflection of space coordinates are given, as

$$X_i \rightarrow X'_i = -X_i; X'_0 = X_0; \\ W_\alpha(X) \rightarrow W'_\alpha(-X_i, X_0) = (\gamma_4 W(-X_i, X_0))_\alpha, \\ \gamma_4 W_{\pm,s}(v_i = 0) = \rho_3 W_{\pm,s}(v_i = 0) = \pm W_{\pm,s}(v_i = 0). \quad (6)$$

The above fact (iii) means that the $W_{+,s}$ and $W_{-,s}$ form a parity-doublet, while the fact (ii) implies that (generator of) chiral transformation $-\gamma_5$ transforms the members of this doublet each other.

Above we have given only the rule for relevant transformations of fundamental vectors $W_\alpha^{(\pm)}$'s in the $\tilde{U}(4)_{DS}$ -tensor space. The rules of any transformation on the spin-flavor WF of composite hadrons, tensors in $\tilde{U}(12)$ -space, are derived from those of constituents as fundamental vectors of this space.

In this connection we note especially, concerning our relevant H/L meson system, that the static $U(12)$ symmetry expected on the constituent L-quarks, leads to existence of parity-doublets (due to (iii)), whose members play mutually the role of chiral partners (due to (ii)).

Finally here, on the basis of above arguments, we give some comments on the conventional approach; applying Dirac equation to light-quarks in the heavy-light (HL) quark meson[16, 17], and on the covariant kinematical framework[18] based on heavy-quark (HQ) effective theory: These preceding works are surely appropriate for introducing HQ symmetry in the framework. However, the framework of Ref.[16, 17] is not covariant, since the whole hadron seems to be at rest ($\mathbf{v} = \mathbf{0}$), and the booster becomes, due to (i), identity. In the work[18] the framework is covariant but missing there the definition of chiral symmetry. The framework of Ref.[18] is equivalent to that of our preceding scheme, the oscillator quark model[19, 21], before introducing the "chiral spinors", $u_{-,s}$ and $v_{-,s}$, into the members of complete set of fundamental vectors $W_\alpha^{(+)}(P)$ and $W_\alpha^{(-)}(P)$.

II. ESSENTIALS AND ORIGIN OF $\tilde{U}(12)$ -CLASSIFICATION SCHEME

Before going into detailed application of our scheme which might be unfamiliar to most readers, we shall describe the essential points and the origin of the $\tilde{U}(12)$

TABLE I: Static $U(12)$ symmetry and its covariant representation-space for light quarks

Symmetry	Representation Space
hadron at rest ($\mathbf{P} = 0$)	hadron in moving frame (\mathbf{P})
$U(12)_{SF} \times O(3)_L$	$\tilde{U}(12)_{SF} \times O(3, 1)_L$
$U(12)_{SF} \supset SU(6)_{SF} \times SU(2)_\rho,$	
$\supset SU(3)_L \times SU(3)_R \times SU(2)_\sigma$	

scheme, which has a long history. (As for details, see our review articles[20] and the original ones[11].)

A. Covariant $\tilde{U}(12)$ -classification scheme and static $U(12)$ symmetry

The framework of $\tilde{U}(12)$ -classification scheme is manifestly Lorentz-covariant, and the hadron wave functions (WF) are supposed to be generally tensors in the $\tilde{U}(12)_{SF} \times O(3, 1)_L$ space, where the $\tilde{U}(12)_{SF} \supset U(3)_F \times \tilde{U}(4)_{DS}$.

Here it is to be noted that we do not assume the $\tilde{U}(12)$ symmetry or any other rigorous relativistic symmetry (including the generators of Lorentz transformation $\Sigma_{\mu\nu}$ as symmetry generators), leading to such an irreducible representation as containing its members with different spins. There is no such type of relativistic symmetries because of No-Go theorem by Coleman and Mandula[22]. Instead we mean by the term of $\tilde{U}(12)$ -classification scheme that all members belonging to the same multiplet have the same *squared-mass in the ideal limit* (, where are neglected the effects of perturbative QCD and those due to the spontaneously-broken chiral symmetry) and that strong interactions are consistent with a unitary group of the $U(12)_{SF}$ symmetry, when all relevant hadrons are at rest. In this sense our scheme is, more strictly, to be called the static $U(12)$ symmetry scheme. Because of this rest condition[23], No-Go theorem is not applicable to our classification scheme, since static $U(12)$ does not include $\Sigma_{\mu\nu}$ as its generators[24]. (As for details, see the appendix.)

In our scheme the representation space for light constituent quarks is extended from the non-relativistic (NR) $SU(6)_{SF} \times O(3)_L$ space to the covariant $\tilde{U}(12)_{SF} \times O(3, 1)_L$ space: This implies the introduction of new additional $SU(2)$ space on the ρ -spin and mean the extension of conventional $SU(6)_{SF}$ to $U(12) \supset SU(6)_{SF} \times SU(2)_\rho$. In another representation, the static $U(12)$ also includes the conventional, chiral $SU(3)_L \times SU(3)_R \times SU(2)_\sigma$ as a subgroup. Here the ρ -spin and the Pauli's σ -spin are those concerning with the decomposition of Dirac γ matrices, $\gamma \equiv \sigma \otimes \rho$. In the Pauli-Dirac representation the direction of ρ_3 -spin denotes that of time-flow in the rest frame of relevant hadron. These situations are shown in table I.

B. Origin of $\tilde{U}(12)$ -classification scheme

The origin of $\tilde{U}(12)$ -classification scheme is traced back long ago. In 1965, shortly after the proposal of quark model and, successively, of the $SU(6)_{SF}$ symmetry, Salam et al. and Sakita-Wali independently proposed[26] the $\tilde{U}(12)$ symmetry as a relativistic extension of the $SU(6)_{SF}$ symmetry. However, it is now well-known that its relativistic extension as a rigorous mathematical group is impossible, as was mentioned in the preceding subsection.

In 1970 one of the authors proposed the urciton scheme[21], which is the direct origin of our new scheme, for the purpose of treating multi-quark hadrons systematically and covariantly. Here the constituent quarks are regarded as excitons related to the relevant hadron, and the hadron WF correspond to the Fock amplitudes for the system of multi-exciton quarks, moving with the same velocity as the relevant hadron. As a result we can show that the hadron WF reproduces the successful contents (that is, the results of $SU(6)_{SF}$) of the original $\tilde{U}(12)$ symmetry[26]. (See, Appendix 3.) The WF in this scheme is manifestly covariant. However, we can not yet treat the chiral symmetry, since it was assumed there that only “boosted-Pauli spinors” are applied as physical ones (following the original $\tilde{U}(12)$ -papers[26]). Our new $\tilde{U}(12)$ -classification scheme is developed only by discarding this now-unnecessary restriction and by taking into account all elements of the complete set to be physical states in expanding $\tilde{U}(12)$ (tensor)-space.

C. Fundamental representation of confined-quarks and chiral states/chiralons

Because of the new freedom $SU(2)_\rho$, our scheme becomes reconcilable with the chiral symmetry, and we are led to existence of new states for hadrons (named chiral states), which are out of the conventional NR scheme. The fundamental vectors in $\tilde{U}(4)_{DS}$ tensor-space are given by the original Dirac spinors, $W_\alpha^{(\pm)}(P)$ and its Pauli-conjugate $\overline{W^{(\pm)}(P)}^\beta \equiv (W^{(\pm)}(P)^\dagger \gamma_4)^\beta$. Here $W_\alpha^{(+)}(P)$ and $W_\alpha^{(-)}(P)$ are the four-dimensional Fourier amplitudes of $W^{(+)}(x)$ and $W^{(-)}(x)$, respectively, which are the positive and negative frequency parts of the solutions of the “local” Klein-Gordon equation $W_\alpha(x)$, x_μ being the space-time coordinate in $O(3, 1)_L$ (see section III C). The complete set of $W^{(+)}(P)$ ($W^{(-)}(P)$) consists of the four elements $u_{r,s}(P)(v_{\bar{r},\bar{s}}(P))$ describing the spinor freedom of confined quarks(anti-quarks) inside of hadrons, where $r(\bar{r}) = \pm$ and $s(\bar{s}) = \pm$ denote eigen values of $\rho_3(\bar{\rho}_3 = -\rho_3^t)$ and $\sigma_3(\bar{\sigma}_3 = -\sigma_3^t)$ for $u(v)$, respectively, and $P_\mu(P_0 \equiv \sqrt{\mathbf{P}^2 + M^2} > 0)$ being the center of mass four momenta of hadrons (not quarks): Conventionally, the spinors $u_{-,s}(v_{-,s})$ for quarks(anti-quarks) are identified with the spinors $v_{+,s}(u_{+,s})$ for anti-quarks(quarks). This is based upon

the hole-theory on the free quark field theory. Correspondingly, in NRQM only the NR two-component Pauli-spinors $\chi_s(\chi_{\bar{s}})$ for quarks(anti-quarks), which becomes equivalent to the upper(lower) two-components of four-component boosted-Pauli spinors $u_{+,s}(v_{+,\bar{s}})$ in the static limit with $\mathbf{P} = \mathbf{0}$, are applied. In the original $\tilde{U}(12)$ -symmetry scheme (, and in the previous framework of covariant oscillator quark model(COQM)[19],) only the boosted-Pauli spinors $u_{+,s}(v_{+,\bar{s}})$ are regarded as physical states.

However, the above picture on hole theory and the identification of $u_{-,s} = v_{+,\bar{s}}$ ($v_{-,\bar{s}} = u_{+,s}$) is only applicable to the free quarks (or to whole free-hadrons), and not separately to the confined constituent-quarks, coexisting with the other quarks. This is so, because the application of hole-theory implies that all quantum numbers of particle-hole are to be replaced by their conjugates. Especially, concerning the color freedom, the application induces the change of $\mathbf{3}_c$ of quark-holes to the $\mathbf{3}_c^*$ of anti-quarks, leading to the violation of color-singlet condition for the relevant hadron. Accordingly, in describing the spinor WF of composite hadrons covariantly, all four Dirac spinors, $u_{\pm,\pm}$ and $v_{\pm,\pm}$, respectively, for quarks and for anti-quarks, are required as the elements of complete set of the fundamental vectors in $\tilde{U}(4)_{DS}$ -space.

As is described above, our Dirac spinors as the fundamental vectors in $\tilde{U}(4)_{DS}$ (tensor)-space is representing some mathematical quantity, to be appropriately called “exciton quarks”, simulating the properties of constituent quarks inside hadrons, and different from those representing constituent-quarks in the dynamical composite models. We shall call our Dirac spinors as the ur-citon spinor. The notion of exciton quarks (and its name urciton) was first introduced in the paper[21] long ago, and the urciton spinor seems to be of the similar nature to the u-spinor, introduced in “the 144-fold way out” from the trouble of relativistic $SU(6)$ [27]. (See, Appendix A.6.)

These urciton spinors are transformed with each other by operating $-\gamma_5$, the generator of chiral transformation, as $u_{+,s} \leftrightarrow u_{-,s}$, $v_{+,\bar{s}} \leftrightarrow v_{-,\bar{s}}$, and play mutually a role of chiral partners. For later convenience we call the spinors $u_{+,s}$ and $v_{+,\bar{s}}$ the (relativistic) Pauli-spinors ; while do $u_{-,s}$ and $v_{-,\bar{s}}$ the chiral-spinors (see, III as for the strict formula).

The above mentioned existence of new states for hadrons, to be called chiral states, is due to this introduction of $u_{-,s}$ and $v_{-,\bar{s}}$ in representing the confined quarks inside of hadrons: The chiral states of hadrons are defined as being represented by the tensors containing at least one $\tilde{U}(4)_{DS}$ -index of chiral-spinors ,while we define, the Pauli states of hadrons as being described by those of only Pauli-spinors. We call, especially, the hadrons being represented purely by the Pauli/chiral states as Paulons/chiralons. Here it should be noted that the physical hadrons generally belongs to a superposition of the Pauli- and chiral-states. They, Paulons and chiralons,

form a linear representation of chiral symmetry.

Furthermore, it should be remarked that due to the introduction[20] of chiral states into physical complete set of \mathcal{S} -matrix bases, our $\tilde{U}(12)_{\text{stat}}$ -symmetry classification scheme becomes free from the problem of unitarity[27] in the original $\tilde{U}(12)_{SF}$ -symmetry scheme[26]. (See the Appendix A.6 for details.)

D. Level-structures of light-quark mesons and baryons, and heavy-light quark mesons

The systematic and rather rigorous considerations of the general meson-system in our new scheme have been given in the second paper of Ref.[11], and the level structure of light-quark meson and baryon system have been shortly described in the first paper of Ref.[11]. Here we will recapitulate the essential points and pick up the candidates for chiralons.

The light-light(LL) quark mesons in the ground-state($L = 0$), which are classified as $\mathbf{6} \times \mathbf{6}^* = \mathbf{36}$ in $SU(6)_{SF}$, are assigned as $\mathbf{12} \times \mathbf{12}^* = \mathbf{144}$ in $\tilde{U}(12)_{SF}$. The **144** includes two sets of pseudoscalar and vector nonets, $\{P_s^{(N)}, V^{(N)}; P_s^{(E)}, V^{(E)}\}$ and also two sets of scalar and axial vector nonets, $\{S^{(N)}, A^{(N)}; S^{(E)}, A^{(E)}\}$. The respective members in the two brackets play a role of chiral partners mutually. The π -meson nonet and σ -meson nonet are assigned to the $P_s^{(N)}$ and $S^{(N)}$, respectively, where the former being maximally mixed states of the Pauli- and chiral-states, while the latter being purely-chiral states(that is, chiralons). The conventional ρ -meson nonet is conjectured to be dominantly the Pauli states(that is, Paulons). Concerning the remaining nonets, $P_s^{(E)}$, $V_\mu^{(E)}$ and $S^{(E)}$, $A^{(N)}$, $A^{(E)}$, it is suggestive that the identification of the relevant P -wave states in the conventional scheme is still in some confusion, and that the lower excited-vector and pseudo-scalar states seem to contain some extra-levels.

The light quark (qqq) baryon system in the ground S -wave states is classified as $(\mathbf{12} \times \mathbf{12} \times \mathbf{12})_S = \mathbf{364}$, which includes baryon and anti-baryon. The **182** of baryons is decomposed into $\mathbf{182} = \mathbf{56} + \mathbf{70} + \mathbf{56}'$, where **56** corresponds to the conventional **56** in $SU(6)_{SF}$. Additional **70(56')** with negative(positive) parity have generally very wide widths and are considered to be observed only as backgrounds, except for the cases of the problematic $\Lambda(1405)$ (Roper $N(1440)$). (As for more details, see Appendix A.3.)

Inclusion of heavy quarks (Q) in our covariant $\tilde{U}(12)$ -classification scheme is straightforward: In table I the representation space $\tilde{U}(12)_{SF}$ for the flavor and spinor freedom of light-quarks is to be extended to the $[\tilde{U}(12)_{SF}]_q \otimes [\tilde{U}(4)_{SF}]_Q$ for the heavy-light(HL) quark hadrons, the $[\tilde{U}(4)_{SF}]_Q$ being $[U(1)_F \otimes \tilde{U}(4)_{DS}]_Q$. The symmetry in the rest frame becomes $[U(12)_{SF}]_q \otimes [U(2)_{SF}]_Q$, where the $U(2)_{SF}$ being $U(1)_F \otimes SU(2)_S$, since for the heavy quarks inside of hadrons the heavy

quark spin-symmetry is valid, implying only the boosted Pauli-spinors are required as ur-citon spinors.

The HL quark mesons in the ground state ($L=0$), which are classified as $\mathbf{6} \times \mathbf{2} = \mathbf{12}$ in the non-relativistic $SU(6)_q \times SU(2)_Q$ scheme, are assigned as $\mathbf{12} \times \mathbf{2} = \mathbf{24}$ in the static $U(12)_q \otimes SU(2)_Q$ symmetry scheme (, embedded in the covariant $[\tilde{U}(12)_{SF}]_q \times [\tilde{U}(4)_{SF}]_Q$ representation space). The $\mathbf{24}$ includes the $\mathbf{12}$ of the conventional pseudo-scalar 0^- - and vector 1^- $SU(3)_F$ triplets and the $\mathbf{12}$ of the newly-appearing scalar 0^{+-} - and axial-vector 1^{+-} - $SU(3)_F$ triplets in the $(q\bar{Q})$ system. The formers are Paulons, while the latters are chiralons. Their anti-particles in the $(\bar{q}Q)$ system belong to the multiplet $\mathbf{12}^* \times \mathbf{2} = \mathbf{24}^*$.

III. COVARIANT DESCRIPTION OF HEAVY-LIGHT(HL) QUARK MESONS

In this section we shall review briefly on how to describe covariantly the composite hadrons, in so far as concerned with the HL-quark mesons in the $\tilde{U}(12)$ -classification scheme. Firstly it should be noted that the $\tilde{U}(12)$ -classification scheme is, in the present stage, a mere kinematical framework proposed semi-phenomenologically for describing covariantly composite hadrons, although it is expected to be derived dynamically from non-perturbative treatment of QCD.

A. Attributes and wave functions of composite hadrons

Our relevant HL mesons(, more generally composite hadrons,) should have, as their indispensable attributes, i) definite mass and ii) spin, iii) definite Lorentz-transformation properties, and iv) definite quark-composite structures. Therefore, their wave function(WF) should represent them evidently, and have the symmetry expected for the relevant mesons as a composite system (the attribute iv)), being bound by QCD.

Accordingly, we set up the wave function(WF) of the relevant HL $(Q\bar{q})$ -meson (and LL $(q\bar{q})$ -meson) system as

$$\Phi_A^B(x, y); \quad A = (\alpha, a), \quad B = (\beta, b); \quad (7)$$

where $\alpha, \beta = (1 \sim 4)$; $a, b = (u, d, s)$ and (c, b) for q and \bar{Q} ; $\alpha(\beta)$ denotes the suffix of Dirac spinor of quark(anti-quark). (The color and flavor indices, which are trivial for the relevant problem, are omitted throughout this paper except for necessary places.) In setting this we have imaged as a guide the field theoretical expression for the WF as

$$\Phi_{M,A}^B(x, y) \sim \langle 0 | \psi_A(x) \bar{\psi}^B(y) | M \rangle + \langle M^c | \psi_A(x) \bar{\psi}^B(y) | 0 \rangle, \quad (8)$$

where $\psi_A(\bar{\psi}^B)$ denotes the quark field (its Pauli-conjugate) and $|M\rangle(|M^c\rangle)$ denotes the composite meson

(its charge-conjugate) state. We have also imaged, for WF of its charge-conjugate meson system, the field theoretical expression as

$$\Phi_{M^c,B}^A(y, x) \sim \langle 0 | \psi_B(y) \bar{\psi}^A(x) | M^c \rangle + \langle M | \psi_B(y) \bar{\psi}^A(x) | 0 \rangle, \quad (9)$$

Then, the WF Φ and their Pauli-conjugates $\bar{\Phi}$, (defined by $\bar{\Phi} \equiv \gamma_4 \Phi^\dagger \gamma_4$,) satisfy mutually the relations

$$\begin{aligned} \Phi_{M^c,B}^A(y, x) &= \overline{\Phi_{M,B}^A(x, y)}, \\ \Phi_{M,A}^B(x, y) &= \overline{\Phi_{M^c,A}^B(y, x)}, \end{aligned} \quad (10)$$

These relations imply that the total WF $\Phi_A^B(x, y)$ of the composite meson and its charge conjugate meson system satisfy(, as they should,) the self-conjugate relation :

$$\Phi_A^B(x, y) = \bar{\Phi}_A^B(y, x), \quad (11)$$

where

$$\Phi_A^B(x, y) \equiv \sum_M \Phi_{M,A}^B(x, y) = \sum_{M^c} \Phi_{M^c,B}^A(y, x). \quad (12)$$

B. Klein-Gordon equation

In order to fix the mass of HL-mesons (the first attribute) our WF Φ_A^B are assumed to satisfy the master Klein-Gordon equation of Yukawa-type[12].

$$\left[(\partial/\partial X_\mu)^2 - \mathcal{M}^2(r_\mu) \right] \Phi_A^B(X, r) = 0, \quad (13)$$

where X_μ is the CM coordinate of meson and r_μ is the relative coordinate. Here the squared-mass operator \mathcal{M}^2 is Lorentz-scalar and assumed to be diagonal on and independent of flavor-spinor indices, A and B , of light-quarks in the ideal limit(, neglecting possible effects due to perturbative QCD and vacuum condensate). This assumption leads to the $U(12)_{stat}$ -symmetric *squared-mass* spectra of hadrons in the ideal limit and makes *our scheme reconcilable with the chiral symmetry* concerning the light quarks, as is explained in the sub-section III F. As a concrete model of \mathcal{M}^2 we adopt the covariant oscillator in COQM[33].

C. Internal WF with definite mass and spin

The total WF are separated into the two(positive or negative frequency) parts concerning the CM plane-wave motion (with four-momentum $P_{N,\mu}$), and expanded in terms of mass eigenstates concerning the internal space-time variables, as

$$\begin{aligned}\Phi_A^B(X, r) &= \sum_{N, P_N (P_{N,0} > 0)} [e^{iP_N \cdot X} \Psi_{N,A}^{(+)\beta}(P_N, r) + e^{-iP_N \cdot X} \Psi_{N,A}^{(-)\beta}(P_N, r)], \\ \mathcal{M}^2 \Psi_{N,A}^{(\pm)\beta}(P_N, r) &= M_N^2 \Psi_{N,A}^{(\pm)\beta}(P_N, r).\end{aligned}\tag{14}$$

The internal WF $\Psi_{N,A}^{(\pm)\beta}$ of hadrons with definite mass M_N and total spin $J = L + S$, being tensors in the $\tilde{U}(4)_{D.S.} \times O(3, 1)_L$ space, is given by a relevant linear-combination of direct products of respective subspace eigen-functions, the extended Bargmann-Wigner(BW) spinors $W_A^B(P_N)$ on the $\tilde{U}(12)$ space and the Yukawa oscillator function $O(P_N, r)$ on the $O(3, 1)_L$ space, as

$$\Psi_{J,A}^{(\pm)\beta}(P_N, r) = \sum_{i,j} c_{ij}^J W_A^{(\pm)\beta(i)B}(P_N) O^{(j)}(P_N, r), \tag{15}$$

In this work we have concerned only with the lowest S-wave states of HL-quark mesons. For these states the expansion (15) is not necessary and their internal WF are given as a direct product of the spinor WF (Eqs.(21) and (22)) and the S-wave oscillator-function $O_S(P_N, r)$ (Eq.(23)). For the excited states it is effective to use, as bases of expansion Eq. (15), the WF with definite j_q in HQET. The relation between our LS-bases and those in HQET has been given in Ref.[28]. An investigation along this line of Isgur-Wise function in semi-leptonic decay processes was made in Ref.[29].

The Bargmann-Wigner spinors $W_\alpha^{(\pm)\beta}(P)$, aside from the flavor-indices, are defined as the positive/negative Fourier amplitudes of solutions $W_\alpha^\beta(X)$ of the local Klein-Gordon equation as follows:

$$\begin{aligned}(\frac{\partial^2}{\partial X_\mu^2} - M^2) W_\alpha^\beta(X) &= 0, \\ W_\alpha^\beta(X) &= \sum_{P_\mu (P_0 > 0)} (e^{iP \cdot X} W_\alpha^{(+)\beta}(P) + e^{-iP \cdot X} W_\alpha^{(-)\beta}(P)).\end{aligned}\tag{16}$$

Then the $W_\alpha^{(\pm)\beta}(P)$ are generally given by the bi-products(, such as representing the physical situation of relevant meson system) of the fundamental vectors of $\tilde{U}(4)_{DS}$ space, $W_\alpha^{(\pm)}(P)$ and $\bar{W}^{(\pm)\beta}(P)$, as specified in the following subsection III D.

The urciton Dirac spinors, $W_\alpha^{(+)}(P)$ and $W_\alpha^{(-)}(P)$, are, respectively, the positive and negative frequency Fourier amplitudes of a single-index BW spinors $W_\alpha(X)$. For clarity we give the explicit form of them:

$$W_\alpha^{(+)}(P) = \sum_{s, \bar{s} = \pm} (u_{r=+, s, \alpha}(P) + v_{\bar{r}=-, \bar{s}, \alpha}(P)), \tag{17}$$

$$W_\alpha^{(-)}(P) = \sum_{\bar{s}, s = \pm} (v_{\bar{r}, \bar{s}, \alpha}(P) + u_{r=-, s, \alpha}(P)), \tag{18}$$

$$u_{+, s}(P)_\alpha = \begin{pmatrix} ch\theta \chi^{(s)} \\ sh\theta \mathbf{n} \cdot \boldsymbol{\sigma} \chi^{(s)} \end{pmatrix}_\alpha, \quad u_{-, s}(P)_\alpha = (-\gamma_5)_\alpha^{\alpha'} u_{+, s}(P)_{\alpha'}, \quad (-\gamma_5 = \rho_1), \tag{19}$$

$$\bar{v}_{+, \bar{s}}(P)^\beta = (sh\theta \chi^{(\bar{s})\dagger} \mathbf{n} \cdot \boldsymbol{\sigma}, -ch\theta \chi^{(\bar{s})\dagger})^\beta, \quad \bar{v}_{-, \bar{s}}(P)^\beta = \bar{v}_{+, \bar{s}}(P)^{\beta'} (\gamma_5)_{\beta'}^\beta, \tag{20}$$

where $ch\theta = \sqrt{\frac{E+M}{2M}}$ and $sh\theta = \sqrt{\frac{E-M}{2M}}$. The $r_3(\bar{r}_3)$ is the eigen-value of $\rho_3(v) \equiv -iv \cdot \boldsymbol{\gamma}$ ($\bar{\rho}_3(v) \equiv iv \cdot \boldsymbol{\gamma}$) which reduces to the ordinary $\rho_3(\bar{\rho}_3)$ at the rest frame, where $v_\mu \equiv P_\mu/M = (0, 0, 0, i)$.

The $u_{\pm, s}(P)(\bar{v}_{\pm, s}(P))$ have $r_3 = \pm 1(\bar{r}_3 = \pm 1)$ and (,as was mentioned in II C) the $u_{+, s}(v_{+, \bar{s}})$ are (relativistic) Pauli-spinors, which have their correspondents in NRQM; while the $u_{-, s}(v_{-, \bar{s}})$ are chiral spinors, which have newly appeared in the $\tilde{U}(12)$ -classification scheme.

D. Spinor WF of meson system and chiral transformation

In describing the spinor WF of light-quark (LL) meson system, because of chiral symmetry of the confined light-quarks, the members of BW spinors with all the combinations of $(r_3, \bar{r}_3) = (\pm, \pm)$ are expected to be required in nature. Thus the total $W_\alpha^\beta(v)$ -space is equivalent to all the 16 components of Dirac γ -matrices, and it is expanded by them. Thus, the scalar mesons in σ -nonet, concerning with $(1)_\alpha^\beta = \delta_\alpha^\beta$, are naturally included in the ground-state chiralons.

The states/mesons described with $(r_3, \bar{r}_3) = (+, +)$, which have their correspondents in the conventional NR-classification scheme, are Pauli states/Paulons. The other states/mesons described with $(r_3, \bar{r}_3) = (+, -), (-, +)$ and $(-, -)$, not appearing in NRQM, are chiral states/chiralons, since these states are obtained through the chiral $-\gamma_5$ -transformation of Pauli states/Paulons, where the value of r_3 and \bar{r}_3 changes as $+\rightarrow -$. They, Paulons and chiralons, form together a linear representation of chiral symmetry.

In the heavy-light (HL) meson system the states with $(r_3, \bar{r}_3) = (+, +)$ and $(+, -)$ are expected to be realized,

reflecting the physical situation that the HL meson system has the non-relativistic $SU(2)_s$ spin symmetry (the relativistic, chiral symmetry) concerning the constituent Heavy quarks (Light quarks). Accordingly the scalar S and axial-vector A_μ chiralons with $(r_3, \bar{r}_3) = (+, -)$, as well as the pseudo-scalar P_s and vector V_μ Paulons with $(r_3, \bar{r}_3) = (+, +)$ are predicted to exist.

The explicit form of $W_\alpha^{(+)\beta}(v)$ and $W_\alpha^{(-)\beta}(v)$ for $D(c\bar{q})$ -mesons is given as follows (where $\tilde{\gamma}_\mu \equiv \gamma_\mu - v_\mu(v \cdot \gamma)$ satisfying $P_\mu \tilde{\gamma}_\mu = 0$);

$$\begin{aligned} W_\alpha^{(+)\beta}(v) &= \left(U_\alpha^{(+)\beta}(v), C_\alpha^{(+)\beta}(v) \right) \\ U_\alpha^{(+)\beta}(v) &\equiv \sum_{s, \bar{s}} u_{+,s,\alpha}^{(c)}(P) \bar{v}_{(\bar{q})+, \bar{s}}^\beta(P) = \left(\frac{1 - iv \cdot \gamma}{2\sqrt{2}} [i\gamma_5 D(P) + i\tilde{\gamma}_\mu D_\mu(P)] \right)_\alpha^\beta, \\ C_\alpha^{(+)\beta}(v) &\equiv \sum_{s, \bar{s}} u_{+,s,\alpha}^{(c)}(P) \bar{v}_{(\bar{q})-, \bar{s}}^\beta(P) = \left(\frac{1 - iv \cdot \gamma}{2\sqrt{2}} [D_0^\chi(P) + i\gamma_5 \tilde{\gamma}_\mu D_\mu^\chi(P)] \right)_\alpha^\beta, \end{aligned} \quad (21)$$

$$\begin{aligned} W_\alpha^{(-)\beta}(v) &= \left(U_\alpha^{(-)\beta}(v), C_\alpha^{(-)\beta}(v) \right) \\ U_\alpha^{(-)\beta}(v) &\equiv \sum_{s, \bar{s}} v_{+, \bar{s}, \alpha}^{(\bar{c})}(P) \bar{u}_{(q)+, s}^\beta(P) = \left(\frac{1 + iv \cdot \gamma}{2\sqrt{2}} [i\gamma_5 \bar{D}^\dagger(P) + i\tilde{\gamma}_\mu \bar{D}_\mu^\dagger(P)] \right)_\alpha^\beta, \\ C_\alpha^{(-)\beta}(v) &\equiv \sum_{s, \bar{s}} v_{+, \bar{s}, \alpha}^{(\bar{c})}(P) \bar{u}_{(q)-, s}^\beta(P) = \left(\frac{1 + iv \cdot \gamma}{2\sqrt{2}} [\bar{D}_0^{\chi, \dagger}(P) + i\gamma_5 \tilde{\gamma}_\mu \bar{D}_\mu^{\chi, \dagger}(P)] \right)_\alpha^\beta. \end{aligned} \quad (22)$$

In Eq.(21) and Eq.(22) $U_\alpha^{(+)\beta}(v)$ ($C_\alpha^{(+)\beta}(v)$) are represented in terms the positive-frequency part of pseudoscalar D and vector D_μ (scalar D_0^χ and axial-vector D_μ^χ), while $U_\alpha^{(-)\beta}(v)$ ($C_\alpha^{(-)\beta}(v)$) are represented in terms of the negative-frequency part of the respective $\bar{D}(q\bar{c})$ -mesons. From Eq.(21) and Eq.(22) it is evident that Paulons (D, D_μ) are transformed into chiralons (D_0^χ, D_μ^χ) through the chiral transformation of light anti-quark,

$W(v) \rightarrow W(v) e^{i\gamma_5 \frac{\lambda^a \alpha^a}{2}}$ (λ^a being the flavor $U(3)_F$ -matrices).

E. Bi-spinor field of $D(c\bar{q})$ -meson system

The bi-spinor field of $D(c\bar{q})$ -meson ($(\bar{q} = \bar{u}, \bar{d}, \bar{s})$ in the $(c\bar{q})$ -system) denoted as $\Phi_D(X, r)$, is defined by

$$\begin{aligned} \Phi_{D,A}^B(X, r) &= \int \frac{d^3\mathbf{P}}{\sqrt{(2\pi)^3 2E}} \left(W_{D,A}^{(+)\beta}(v) e^{iP \cdot X} + W_{D,A}^{(-)\beta}(v) e^{-iP \cdot X} \right) O(P, r) \\ &= \frac{1}{2\sqrt{2}} \left(1 - \frac{\gamma \cdot \partial}{\sqrt{\square}} \right) \left(i\gamma_5 \phi_{D,a}^b + i\gamma_\mu \phi_{D_\mu,a}^b + \phi_{D_0^\chi,a}^b + i\gamma_5 \gamma_\mu \phi_{D_\mu^\chi,a}^b \right)_\alpha^\beta O(P, r), \end{aligned} \quad (23)$$

where the $\phi_R(X)$ ($R = D, D_\mu, D_0^\chi, D_\mu^\chi$) represent the local D -meson field in the $(c\bar{q})$ system. The $\bar{D}(q\bar{c})$ -

meson field is given by its Pauli-conjugate. $\Phi_{\bar{D}}(X, r) =$

$\bar{\Phi}_D(X, r) = \gamma_4 \Phi_D(X, r)^\dagger \gamma_4$. The $\Phi_{\bar{D}}$ is obtained from Φ_D by replacing $W^{(\pm)}(v)$ with $\bar{W}^{(\pm)}(v) (\equiv \overline{W^{(\mp)}}(v))$, and becomes $\Phi_{\bar{D}} = \bar{\Phi}_D$ due to Eq.(10).

F. Static $U(12)$ symmetry embedded in $\tilde{U}(12)$ space of representation

The equation (13), using the abbreviated notation $\square \equiv (\partial/\partial X_\mu)^2$ and $\gamma \cdot \partial \equiv \gamma_\mu \cdot \partial_{X\mu}$, is derived from

$$\mathcal{S} = \int d^4X d^4r \mathcal{L}(X, r), \quad \mathcal{L}(X, r) \equiv \langle \Phi_D(X, r) (\partial_X^2 - \mathcal{M}^2(r)) \vec{F}_U(X) \bar{\Phi}_D(X, r) \rangle, \quad (24)$$

where the notation $\langle M \rangle$ denotes to take the trace on the spinor-flavor indices, and a factor, to be named as unitarizer, is inserted between the trace on light-quarks. In Eq.(24), through the integration, this factor $\vec{F}_U(X) \equiv \frac{\gamma \cdot \vec{\partial}}{\sqrt{\square}}$ and $\bar{F}_U(X) \equiv \frac{\gamma \cdot \vec{\partial}}{\sqrt{\square}}$, becomes $F_U(v) = \mp i v \cdot \gamma$ for

the following action as

$\bar{W}^{(\pm)}(v)$, leading to the change of signs, $\bar{U}^{(\pm)} \rightarrow -\bar{U}^{(\pm)}$ and $\bar{C}^{(\pm)} \rightarrow \bar{C}^{(\pm)}$, which makes the \mathcal{S} chiral-invariant. It reduces, in the meson rest frame, to $\mp \gamma_4$, and the overlapping changes such as

$$\begin{aligned} \langle W^{(+)}(v) \mathcal{O}(-iv \cdot \gamma) \bar{W}^{(-)}(v) \rangle &\sim \langle u_c(P) \bar{v}_{\bar{q}}(\pm P) \mathcal{O}(-iv \cdot \gamma) v_{\bar{q}}(\pm P) \bar{u}_c(P) \rangle \\ &\rightarrow \langle u_c(\mathbf{0}) v_{\bar{q}}(\pm \mathbf{0})^\dagger \mathcal{O} v_{\bar{q}}(\pm \mathbf{0}) \bar{u}_c(\mathbf{0}) \cdot \rangle, \end{aligned} \quad (25)$$

where $\mathcal{O} \equiv (\partial_X^2 - \mathcal{M}^2(r))/\sqrt{\square}$ is the scalar Klein-Gordon operator. The last line of Eq. (25) is invariant under the static $U(12)$ transformation, $v_{\bar{q}}(\mathbf{0}) \rightarrow e^{i \frac{\lambda^a}{2} \Gamma_i \alpha_i^a} v_{\bar{q}}(\mathbf{0})$, where λ^a are flavor $U(3)$ matrices, Γ_i are the hermitian Dirac 16 matrices and α_i^a are transformation parameters.

Accordingly the action Eq. (24) leads, after integrating on the d^4r , to the action with the bilinear terms of a series of the local $D^{(N)}(c\bar{q})$ -meson systems with degenerate mass M_N , in conformity with the static $U(12)$ symmetry, as (denoting only the ground state mesons)

$$\mathcal{S}_{\text{stat } U(12)} = - \int d^4X \sum_{R=D, D_0^\chi, D_\nu, D_\nu^\chi} \left[\partial_\mu \phi_R(X) \partial_\mu \phi_R^\dagger(X) + \phi_R(X) M^2 \phi_R^\dagger(X) \right]. \quad (26)$$

Here, it may be worthwhile to note that, in the case of $S_{\tilde{U}(12)}$ (without the above factor $F_U(X)$ in Eq.(24), the Paulon and chiralon-terms in Eq.(26) obtain the oposite signs each other. The action Eq. (24) is used for leading to the conserved electromagnetic current[30] in section V.

IV. MASS SPECTRA FOR GROUND HL-QUARK MESONS

In the $\tilde{U}(12)$ -classification scheme the global mass spectra of quark- and anti-quark mesons in the low-mass region are to be given generally for both Pauli and chiral states, by

$$M_N^2 = M_0^2 + N\Omega, \quad N \equiv 2n + L, \quad (27)$$

(where Ω denotes inverse Regge slope,) taking into account to reproduce the phenomenologically well-known Regge trajectories for Pauli-states: In the relevant HL-quark meson systems all ground-state Paulons (P_s, V_μ) and chiralons (S, A_μ) are degenerate in the ideal limit, and they are expected to split with each others between chiral partners (spin partners) by the spontaneous breaking of the chiral symmetry (the perturbative QCD spin-spin interaction); From the approximate chiral symmetry/HQS regarding the light/heavy

quark constituents we can derive the common relations through the $D(c\bar{q})$ - and $B(b\bar{q})$ -meson systems in the $SU(3)$ limit[32, 34].

$$\begin{aligned}\Delta M^\chi(Q\bar{q}) &\equiv M(0^+) - M(0^-) \\ &= M(1^+) - M(1^-) . \quad (Q = (c, b).) \quad (28)\end{aligned}$$

The value $\Delta M^\chi(Q\bar{s})$ is determined from the experimental mass values of Paulons ($D_s(0^-)$ and $D_s^*(1^-)$) and chiralons ($D_{s0}^\chi(0^+)$ and $D_{s1}^\chi(1^+)$), as

$$\begin{aligned}\Delta M_{J=0}^\chi(Q\bar{s}) &= M_{(c\bar{s};0^+)}(2317) - M_{(c\bar{s};0^-)}(1968) = 349.2\text{MeV}, \\ \simeq \Delta M_{J=1}^\chi(Q\bar{s}) &= M_{(c\bar{s};1^+)}(2459) - M_{(c\bar{s};1^-)}(2112) = 347.2\text{MeV}, \\ \rightarrow \Delta M^\chi(Q\bar{s}) &\simeq 348\text{MeV}.\end{aligned} \quad (29)$$

In Ref.[9], the same value is also applied to the splitting in $Q\bar{n}$ system as $\Delta M^\chi(Q\bar{n}) = \Delta M^\chi(Q\bar{s})$. Here we consider another possibility, taking

$$\begin{aligned}\Delta M^\chi(Q\bar{n}) &= \Delta M^\chi(Q\bar{s}) \frac{a}{b} \\ &= 348 \times \frac{1}{1.44} = 242\text{MeV},\end{aligned} \quad (30)$$

where $a \propto \langle u\bar{u} \rangle_{\text{vac}} = \langle d\bar{d} \rangle_{\text{vac}}$, $b \propto \langle s\bar{s} \rangle_{\text{vac}}$ ($\langle u\bar{u} \rangle_{\text{vac}}$ being the quark bilinear scalar condensates), and we use the values determined by the $SU(3)$ linear σ model(L σ M)[35, 36]. The Eq. (30) is derived from the $SU(3)$ chiral-symmetric Yukawa interaction in the next section.

By using the values Eqs.(29) and (30) and the experimental masses of the ground Pauli-states (P_s, V_μ), we can predict the masses of all the ground-state ($L = 0$) chiralons (S, A_μ) in HL-quark meson systems as given in table II. As is stated in the introduction, the ground S -wave chiral $0^+(D_0^\chi)$ and $1^+(D_1^\chi)$ mesons, to be discriminated from the ordinary P -wave $0^+(D_0^*)$ and $1^+(D_1)$ mesons with $j_q^P = \frac{1}{2}^+$, exist in our scheme. However, for the $(c\bar{n})$ -system, in the present experiments, supposing only one $0^+(1^+)$ meson with broad width exists, the two different values of mass and width for respective mesons are reported: For 0^+ (M, Γ) = (2308 \pm 36, 276 \pm 66)MeV[41, 42], (2405 \pm 28, 262 \pm 37)MeV[43, 44]. For 1^+ (M, Γ) = (2427 \pm 36, 384 \pm 117)MeV[41, 46], (2461 \pm 50, 290 \pm 100)MeV[45, 47]. The mass of $D_0^*(D_1)$ seems to be inconsistent(somewhat different) in two experiments, suggesting that the $D_0^*(2308 \sim 2405)$ ($D_1(2427 \sim 2461)$) is a superposition of two 0^+ (1^+) resonances, one is chiral S -wave $D_0^\chi(2110)$ ($D_1^\chi(2250)$) and the other is P -wave $D_0^*(2400)$ ($D_1(2470)$), (of which masses are predictions by NRQM[37]). The reanalyses of experimental data from this viewpoint are required.

Similarly for the $(c\bar{s})$ system, the chiral S -wave mesons $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$, in addition to the P -wave Pauli states $D_{s0}^*(2466)$ [38] and $D_{s1}(2536)$ [38], are pre-

dicted to exist with broad width, a few hundred MeV, since they have the OZI-allowed DK and D^*K open channels.

From Table II we are able to read the following interesting facts on the hyperfine splittings $\Delta M^{HF}(Q\bar{q})$ between the members with the same quark configurations

$$\Delta M^{HF} \equiv M(1^-) - M(0^-) = M(1^+) - M(0^+), \quad (31)$$

which is derived from Eq. (28);

ΔM^{HF} is inversely proportional to the heavy-quark masses, and independent of the light-quark masses;

$$\begin{aligned}\Delta M^{HF}(b\bar{q})/\Delta M^{HF}(c\bar{q}) &= 0.047/0.14 = 0.34 \\ \approx m_c/m_b (= m_{J/\psi}/m_\Upsilon) &= 3.1/9.5 = 0.33.\end{aligned} \quad (32)$$

ΔM^{HF} (with the same heavy-quark) is independent of light-quark masses;

$$\begin{aligned}\Delta M^{HF}(c\bar{n}) &\approx \Delta M^{HF}(c\bar{s}) \approx 140\text{MeV}, \\ \Delta M^{HF}(b\bar{n}) &\approx \Delta M^{HF}(b\bar{s}) \approx 50\text{MeV}.\end{aligned} \quad (33)$$

These facts are reasonably understood from the physical situation in the HL-quark meson systems that the heavy-quark (light-quark) behaves non-relativistically (relativistically), leading to the HQ symmetry(chiral symmetry) concerning the heavy-(light-) constituent quarks.

The recently observed new $c\bar{s}$ -meson $D_{sJ}(2632)$ [39] causes another serious problem[40] in the conventional classification scheme. From its decay modes $D_s\eta$ and D^0K^+ , the most natural quantum number is $J^P = 1^-$. However, its mass seems to be too light to be assigned as radially excited 2^3S_1 state in NRQM. In $\tilde{U}(12)$ -classification scheme, $D_{sJ}(2632)$ is naturally assigned as a P -wave chiral $c\bar{s}$ -meson with 1^- . If this is the case, the other P -wave chiral mesons are expected to exist in this mass region $\sim 2600\text{MeV}$. The masses of all these P -wave chiral $c\bar{q}$ -mesons, predicted by using simple assumptions, are given in Table II.

TABLE II: Mass spectra of ground-state($L = 0$) D_s , D , B_s and B meson systems: For D_s and D systems, the masses with $L = 1$ states are also shown. The masses of Paulons, D_0^* and D_1 (D_{s0}^* and D_{s1}), are the predictions by Ref.[37](Ref.[38]). The recently observed $D_{sJ}(2632)$ [39] state is naturally assigned as P -wave 1^- chiral state with $j_q^P = \frac{1}{2}^-$ or $\frac{3}{2}^-$. (Tentatively $\frac{3}{2}^-$ case is chosen in the table.) The chiral 1^- D_n meson mass $M_{D(1-)}$ is predicted by using simple assumption, $M_{D(1-)} = M_{D_1(2422)} + (a/b)(M_{D_{sJ}(2632)} - M_{D_{s1}(2535)})$, with $a/b = 1/1.44$. The masses of the other P -wave $c\bar{q}$ chiral states are given by assuming the spin-dependent splittings being the same as those for P -wave $c\bar{s}$ Pauli states.

$c\bar{s}$ -meson				$c\bar{n}$ -meson				
L	Paulons		chiralons		Paulons		chiralons	
0	0^-	$D_s(1968)$	0^+	$D_{s0}^X(2317)$	0^-	$D(1870)$	0^+	$D_0^X(2110)$
	1^-	$D_s^*(2112)$	1^+	$D_{s1}^X(2459)$	1^-	$D^*(2010)$	1^+	$D_1^X(2250)$
1	0^+	$D_{s0}^*(2466)[38]$	0^-	$D_s^X(2563)$	0^+	$D_0^*(2400)[37]$	0^-	$D^X(2420)$
	1^+	$D_{s1}(2536)[38]$	1^-	$D_s^X(2633)$	1^+	$D_1(2470)[37]$	1^-	$D^X(2490)$
	1^+	$D_{s1}(2535)$	1^-	$D_{sJ}^X(2632)[39]$	1^+	$D_1(2422)$	1^-	$D^X(2490)$
	2^+	$D_{s2}^*(2572)$	2^-	$D_s^X(2669)$	2^+	$D_2^*(2459)$	2^-	$D^X(2525)$
<hr/>								
L	$b\bar{s}$ -meson				$b\bar{n}$ -meson			
0	0^-	$B_s(5369)$	0^+	$B_{s0}^X(5717)$	0^-	$B(5279)$	0^+	$B_0^X(5520)$
	1^-	$B_s^*(5415)$	1^+	$B_{s1}^X(5760)$	1^-	$B^*(5325)$	1^+	$B_1^X(5565)$
<hr/>								

V. DECAY PROPERTIES OF D_s -MESONS

A. Pionic decays

In order to estimate the absolute magnitude of the width of the observed pionic decays $D_{sJ}^*(2317) \rightarrow D_s(1968) + \pi^0$ and $D_{sJ}(2460) \rightarrow D_s^*(2112) + \pi^0$, we shall set up the chiral symmetric effective interaction Lagrangian[48]

$$\begin{aligned} \mathcal{L}_{ND} &= -g_{ND} \langle \Phi_D(X) M(X) \Phi_{\bar{D}}(X) \rangle \\ \mathcal{L}_A &= \frac{g_A + g'_A}{2} \langle \Phi_D(X) (M(X) \overleftrightarrow{\partial}_\mu \gamma_\mu) \vec{F}_U(X) \Phi_{\bar{D}}(X) \rangle \\ &\quad + \frac{g_A - g'_A}{2} \langle \Phi_D(X) \overleftarrow{F}_U(X) (\gamma_\mu \partial_\mu M(X)) \Phi_{\bar{D}}(X) \rangle, \end{aligned} \quad (34)$$

where only the Yukawa interaction of the scalar and pseudo-scalar nonets, $M \equiv s - i\gamma_5 \phi$ ($s \equiv s^a \lambda^a / \sqrt{2}$, $\phi \equiv \phi^a \lambda^a / \sqrt{2}$) as external fields with the light constituent quarks in the HL-meson is taken into account.

The interaction Eq. (34) consists of the three parts: Firstly the g_{ND} term (Yukawa interaction in non-derivative form) gives dominant (compared to the g_A term) contribution to the (quark-) spin non-flip processes. In spontaneous breaking of chiral symmetry, s takes the vacuum expectation value $\langle s \rangle_0 = \text{diag}\{a, a, b\}$ which induces the mass-splittings between chiral partners through the equation $\Delta M^\chi(c\bar{n}) = 2g_{ND}a$ and $\Delta M^\chi(c\bar{s}) = 2g_{ND}b$. By using SU(3)LSM[35, 36], the a and b are related with the pion and kaon decay constants as $b = (2f_K - f_\pi)/\sqrt{2}$, $a = f_\pi/\sqrt{2}$. From this relation and by using the experimental value of $\Delta M^\chi(c\bar{s}) = 348\text{MeV}$, Eq. (29), the g_{ND} is determined as $g_{ND} = 1.84$ giving $\Delta M^\chi(c\bar{n}) = 242\text{MeV}$, which is used for predicting the

masses of chiral D^X mesons in section IV.

Secondly the g_A interaction concerns dominantly (compared to the g_{ND} term) to the spin-flip processes, $D^{*+} \rightarrow$

TABLE III: Formula of pionic decay width Γ of D^{*+} , $D_{s0,1}^X$ and $D_{s0,1}^X$. The $M(M')$ is the mass of initial(final) HL-meson, $\omega = -v \cdot v'$ and $\sin^2\theta$ is the η - π^0 mixing parameter. The f_A , f'_A and f_A^s are defined in the text.

process	Γ
$D^{*+} \rightarrow D^0 \pi^+$	$\frac{ \mathbf{p} ^3}{6\pi M M'} \left(\frac{M+M'}{4a} \right)^2 \left(f_A + \frac{2g_{ND}a}{M+M'} \right)^2$
$D_0^X \rightarrow D\pi, D_1^X \rightarrow D^* \pi$	$\frac{3M' \mathbf{p} }{4\pi M} \left(\frac{1+\omega}{2} \right)^2 g_{ND}^2 (1-f_A')^2$
$D_{s0}^X \rightarrow D_s \pi^0, D_{s1}^X \rightarrow D_s^* \pi^0$	$\frac{M' \mathbf{p} }{2\pi M} \left(\frac{1+\omega}{2} \right)^2 g_{ND}^2 (1-f_A^s)^2 \sin^2\theta$

$D^0 \pi^+$. Thirdly the g'_A interaction concerns the spin non-flip processes, $D_0^X \rightarrow D\pi$ and $D_1^X \rightarrow D^* \pi$.

The formula of the relevant pionic decay widths of $D(c\bar{n})$ and $D_s(c\bar{s})$ mesons, which are derived from Eq. (34), are collected in Table III. The f_A , f'_A and f_A^s are the coefficients of the axial-current of $i\gamma_5 \gamma_\mu$ -type. They are related with g_A and g'_A by $g_A = \frac{f_A}{2a}$ and $g'_A = \frac{f'_A}{2a} = \frac{f_A^s}{2b}$, where the SU(3)-breaking effect is taken into account through the difference of vacuum expectation values $b/a = 1.44$.

The decays of $D_{s0}^X \rightarrow D_s \pi^0$ and $D_{s1}^X \rightarrow D_s^* \pi^0$ are iso-spin violating, and considered to occur by the mixing of intermediate η meson with π^0 meson. We can estimate phenomenologically the value of mixing parameter $\sin^2\theta$, by using the experimental branching ratio[1] of $D_n(c\bar{n})$ meson to the iso-spin violating decay channel as

$$(\sin\theta)_{\text{exp}}^2 \approx \frac{Br(D_s^{*+} \rightarrow D_s^+ \pi^0)(M_{D_s^*}^2/q^3)}{Br(D_s^{*+} \rightarrow D_s^+ \gamma)} / \frac{2Br(D^{*+} \rightarrow D^+ \pi^0)(M_{D^*}^2/q^3)}{Br(D^{*+} \rightarrow D^+ \gamma)} = (0.9 \pm 0.4) \cdot 10^{-3}, \quad (35)$$

which seems to be reasonable order of magnitude due to the virtual EM-interaction.

The experimental value[1] of $\Gamma(D^{*+} \rightarrow D^0 \pi^+) = (96 \pm 22)\text{keV} \times 0.677 = (65 \pm 15)\text{keV}$ is reproduced by $f_A = 0.521$ [49], which corresponds to $g_A (= f_A/(2a)) = 3.96\text{GeV}^{-1}$.

For fixing the value of g'_A , we consider two characteristic models:

- i) $i\gamma_5\gamma_\mu$ -model: $g'_A = -g_A$ [50], where the strength of $i\gamma_5\gamma_\mu$ coupling is common to $D^* \rightarrow D\pi$ and $D_0^X \rightarrow D\pi$.
- ii) $\gamma_5\sigma_{\mu\nu}$ -model: This model starts from the effective Lagrangian,

$$\mathcal{L}_{AX} = g_{AX} \langle W^{(+)}(v) [iq_\mu + \sigma_{\mu\nu}(P + P')_\nu] (-iq_\mu) M(q) \bar{W}^{(-)}(v') \rangle, \quad (36)$$

TABLE IV: The predicted values of pionic decay widths of chiral D_s and D_n mesons.

	$i\gamma_5\gamma_\mu$ -model	$\gamma_5\sigma_{\mu\nu}$ -model
$\Gamma(D_{s,0}^X(2317) \rightarrow D_s \pi^0)$	$381 \pm 168\text{keV}$	$141 \pm 63\text{keV}$
$= \Gamma(D_{s,1}^X(2459) \rightarrow D_s^* \pi^0)$		
$\Gamma(D_0^X(2110) \rightarrow D\pi)$	313MeV	144MeV
$= \Gamma(D_1^X(2250) \rightarrow D^* \pi)$		

which is motivated by the Gordon decomposition of $\bar{v}(P)i\gamma_5\gamma_\mu v(P')$. The \mathcal{L}_{AX} is equivalent to the \mathcal{L}_A , Eq. (34), by taking $g_A = (M + M')g_{AX}$ and $g'_A = -(M - M')g_{AX}$. The experimental $\Gamma(D^{*+} \rightarrow D^0 \pi^+)$ leads to the value $g_{AX} = 1.02\text{GeV}^{-2}$, which gives $f_A = 0.521$ and $f'_A = -0.033$. The effect of f'_A is negligibly small in this model. We predict the values of the relevant pionic decay widths in two model cases in Table IV. The predicted

widths of $D_{sJ}^*(2317)/D_{sJ}(2460)$ are consistent with the experimental values[51] $\Gamma(D_{sJ}^*(2317)) < 4.6\text{MeV}$ and $\Gamma(D_{sJ}(2460)) < 5.5\text{MeV}$.

B. Radiative decay

In order to treat systematically all the radiative transitions between the HL-mesons we shall set up the basic EM-interaction Lagrangian as

$$\begin{aligned} \mathcal{S}_I^{EM} &= \int d^4x_1 d^4x_2 \sum_{i=1,2} j_{i,\mu}(x_1, x_2) A_\mu(x_i) \\ &= \int d^4X \sum_i J_{i,\mu}(X) A_\mu(X), \end{aligned} \quad (37)$$

$$j_{1,\mu}(x_1, x_2) = -id \frac{e_1}{2m_1} \langle \vec{F}_U(x_1) \bar{\Phi} \overleftarrow{F}_U(x_1) [\partial_{1\mu}^\rightarrow - \partial_{1\mu}^\leftarrow + g_M i\sigma_{\mu\nu}^{(1)} (\partial_{1\nu}^\rightarrow + \partial_{1\nu}^\leftarrow)] \Phi \rangle, \quad (38)$$

$$j_{2,\mu}(x_1, x_2) = -id \frac{e_2}{2m_2} \langle \Phi [\partial_{2\mu}^\rightarrow - \partial_{2\mu}^\leftarrow + g_M i\sigma_{\mu\nu}^{(2)} (\partial_{2\nu}^\rightarrow + \partial_{2\nu}^\leftarrow)] \vec{F}_U(x_2) \bar{\Phi} \overleftarrow{F}_U(x_2) \rangle, \quad (39)$$

where

$$d = 2(m_1 + m_2). \quad (40)$$

Our multi-local current $j_{i,\mu}$ is obtained through the “minimal substitution” of $(\partial_{i,\mu} \rightarrow \partial_{i,\mu} - ie_i A_\mu(x_i))$ to our Lagrangian \mathcal{L} (see the footnote [33]). The spin interaction

proportional to g_M is introduced following Ref.[52].

By performing integration on relative space-time coordinates in the first line of the above expression Eq. (37), we obtain the effective heavy- and light-quark current of the HL-mesons:

$$\begin{aligned}
J_{1,\mu} &= \langle (-iv' \cdot \gamma) W^{(-)}(v') \left(e_1(P_\mu + P'_\mu) + d \frac{e_1}{2m_1} g_M i \sigma_{\mu\nu} q_\nu \right) W^{(+)}(v) \rangle. \\
J_{2,\mu} &= \langle W^{(+)}(v) \left((-e_2)(P_\mu + P'_\mu) + d \frac{e_2}{2m_2} g_M i \sigma_{\mu\nu} q_\nu \right) (-iv' \cdot \gamma) \bar{W}^{(-)}(v') \rangle.
\end{aligned} \tag{41}$$

This is one of the most simple forms of the covariant generalization of convection and spin current in NRQM. From Eq. (41) we can easily check that our effective current $J_{i,\mu}(X)$ is conserved in the ideal limit, as it should be.

Our effective current has another remarkable feature due to the covariant nature of our scheme. The spin-current interaction leads to the Hamiltonian

$$\begin{aligned}
\mathcal{H}_i^{spin} &\equiv J_{i,\mu}^{spin} A_\mu \\
&= \mu_i i \sigma_{\mu\nu} q_\nu A_\mu = \mu_i (-i \rho_1 \boldsymbol{\sigma} \cdot \mathbf{E} + \boldsymbol{\sigma}^{(i)} \cdot \mathbf{B}),
\end{aligned} \tag{42}$$

where $\mu_i \equiv d \frac{e_i}{2m_i} g_M$. The \mathcal{H}_i^{spin} contains the interaction through the “intrinsic electric dipole” $-i\mu\rho_1\boldsymbol{\sigma}$ as well as the one through the magnetic dipole $\mu\boldsymbol{\sigma}$. The “intrinsic dipole” gives contributions only for the transitions between chirals and Paulons, while does none for the other transitions.

From the effective currents $J_{i\mu}$ in Eqs. (37) and (41), we can derive the formula of the relevant radiative decay widths, which are given in Table V.

By using Table V we can predict the widths for all the radiative transitions between ground state D_s mesons, which are given in Table VI. We have also shown the comparison with the other models, making reference to Ref.[54].

From the results in Table VI we see that our model gives the much larger widths for γ -transition from chiral to Pauli states (1st and 2nd columns), compared with the other models. Our width is almost the same for transition from Paulon to Paulon (3rd column), while it is the much smaller for transition from chiralon to chiralon (4th column), compared with the prediction by Ref.[9]. This difference comes firstly from the above mentioned feature Eq. (42) of our currents, and secondly from the different identification of the relevant mesons: The narrow D_s mesons are assigned as the conventional P -wave excited states in the other models, while they are the S -wave chiral states other than the P -wave Pauli-states in our scheme.

C. Branching ratios between radiative and pionic decay widths

From the predicted values of pionic and radiative (Table VI) decay widths we obtain the ratios between them. Making reference again to Ref.[54], the results are compared with the other models in Table VII.

As is shown in Table VI, our predicted radiative decay widths of D_{s0}^X and D_{s1}^X is one-order of magnitude larger than the other predictions. Concerning pionic decays, our $\gamma_5\sigma_{\mu\nu}$ -model (where g'_A is negligibly small,) is essentially equivalent to the Ref.[9], where only the g_{ND} and g_A interactions are considered in our language. However, the resulting pionic decay width by Ref.[9] is one-order of magnitude smaller than our prediction, because the isospin violating factor, $\sin^2\theta$, used in Ref.[9] (which is estimated theoretically from the η - π^0 -mixing angle [9, 58, 59] as $\sin^2\theta = \frac{1}{2}\delta_{\eta\pi^0}^2 = 1/(2 \cdot (2 \times 43.7)^2) \simeq 0.65 \times 10^{-4}$), is about one-order of magnitude smaller than our phenomenological estimation in Eq. (35). As a result, the ratios [9] of partial widths of γ -decay to π^0 -decay become similar values to our predictions. Only the Ref.[56] except for us predicts the large pionic decay widths, where another phenomenological estimation of $\sin^2\theta (= \frac{2}{3}\epsilon^2)$ is done; $\frac{2}{3}\epsilon^2 = \frac{2}{3} \frac{\mathcal{B}(\psi(2S) \rightarrow J/\psi \pi^0) p_{\pi^0}^3}{\mathcal{B}(\psi(2S) \rightarrow J/\psi \eta) p_{\pi^0}^3} = \frac{2}{3} ((4.07 \pm 0.47) \times 10^{-2})^2 = (1.11 \pm 0.25) \times 10^{-3}$, which is consistent with our value. The measurements of absolute magnitude of the decay widths of D_{s0}^X and D_{s1}^X are required to clarify the situation.

VI. CONCLUDING REMARKS

Through the investigation of this work it may be concluded that the $D_s(2317)/D_s(2459)$ mesons are shown to be assigned consistently, in the $\tilde{U}(12)$ -classification scheme, as the scalar and axial-vector chirals in the $(c\bar{s})$ ground state. If this is the case, the conventional P -wave scalar and axial-vector mesons, D_{s0}^* and D_{s1}^* , are expected to exist in the higher mass region and decay with wide widths into DK and D^*K , respectively. The $D_{sJ}(2632)$, observed quite recently, is assigned as the P -wave chiral state with $J^P = 1^-$ in the $\tilde{U}(12)$ -classification scheme.

In the $(c\bar{n})$ system two set of 0^+ and 1^+ -mesons are predicted to exist in the lower-mass region, both of which are expected to have wide widths; one is S -wave chirals $D_{0,1}^X$, and the other is ordinary P -wave mesons D_0^* and D_1^* . Recent experimental data of $D\pi(D^*\pi)$ mass spectra show a peak-structure with wide width, which is explained as a single $0^+(1^+)$ meson, $D_0^*(2308 \sim 2405)(D_1(2427 \sim 2461))$. However, this peak-structure is considered to come from the two interfering resonances of D_0^X and D_0^* (D_1^X and D_1^*) in the $\tilde{U}(12)$ -classification scheme, and accordingly the data are necessary to be re-

TABLE V: Formula of radiative decay widths Γ for $c\bar{q}$ mesons: By using $|\mathcal{A}|^2$, the Γ is given by $\Gamma = \frac{\alpha|\mathbf{q}|^3}{2J_I+1} \frac{(M+M')^2}{M^2 M'^2} |\mathcal{A}|^2$, where J_I is the spin of initial meson and $\alpha = 1/137.037$. Choosing the normal quark moment ($g_M = 1$), $\mu_1 = \frac{d}{2} \frac{e_1}{2m_1}$ and $\mu_2 = \frac{d}{2} \frac{e_2}{2m_2}$. $\epsilon = \frac{M-M'}{M+M'}$. The $e_1(-e_2)$ is the charge of the first(second) constituent. For $D_s^+ = c\bar{s}$, $e_1(e_2) = \frac{2}{3}e(-\frac{1}{3}e)$.

	$D_{d,s}^{*+} \rightarrow D_{d,s}^+ \gamma$	$D_{s0}^{\chi} \rightarrow D_s^* \gamma$	$D_{s1}^{\chi} \rightarrow D_s \gamma$	$D_{s1}^{\chi} \rightarrow D_s^* \gamma$	$D_{s1}^{\chi} \rightarrow D_{s0}^{\chi} \gamma$
$ \mathcal{A} ^2$	$(\mu_1 + \mu_2)^2$	$(\mu_1 \epsilon + \mu_2)$	$(\mu_1 \epsilon - \mu_2)$	$2(\mu_1 \epsilon)^2 + 2\mu_2^2$	$(\mu_1 + \mu_2)^2$

TABLE VI: γ -decay widths(keV) of the ground state D_s mesons predicted by various models. π^0 -decay widths are also shown for reference. The constituent quark masses are fixed with the values, $m_u = m_d \equiv m_n = M_\rho/2$, $m_s = M_\phi/2$, $m_c = M_{J/\psi}/2$.

Processes	$i\gamma_5\gamma_\mu$	$\gamma_5\sigma_{\mu\nu}$	BEH[9]	God[53]	CFF[54]	FR[55]	CH[5]	AG[56]
$D_{s0}^{\chi} \rightarrow D_s \pi^0$	381 \pm 169	141 \pm 63	21.5	\simeq 10	7 \pm 1	16	10 \sim 100	129 \pm 43(109 \pm 16)
$D_{s0}^{\chi} \rightarrow D_s^* \gamma$	19.2		1.74	1.9	0.85 \pm 0.05	0.2		\leq 1.4
$D_{s1}^{\chi} \rightarrow D_s^* \pi^0$	381 \pm 169	141 \pm 63	21.5	\simeq 10	7 \pm 1	32		187 \pm 73(7.4 \pm 2.3)
$D_{s1}^{\chi} \rightarrow D_s \gamma$	91.6		5.08	6.2	3.3 \pm 0.6			\leq 5
$D_{s1}^{\chi} \rightarrow D_s^* \gamma$	57.4		4.66	5.5	1.5			
$D^{*+} \rightarrow D^+ \gamma$	1.15		1.63	\longleftrightarrow	1.54 \pm 0.53(experimental value)[1]			
$D_s^* \rightarrow D_s \gamma$	0.33		0.43					
$D_{s1}^{\chi} \rightarrow D_{s0}^{\chi} \gamma$	0.235		2.74					

analyzed along this line.

The radiative decay widths of the relevant $D_{sJ}^*(2317)/D_{sJ}(2460)$ mesons are predicted to be remarkably larger than those estimated in other works in our scheme. These are to be checked experimentally.

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APPENDIX A: $\tilde{U}(12)$ -CLASSIFICATION SCHEME AND STATIC $U(12)$ SYMMETRY

The argument in this paper is based on a covariant classification scheme of hadrons, the $\tilde{U}(12)$ -classification scheme, which was proposed[11] by us several years ago. In this scheme composite hadrons have a new type of symmetry extended from the non-relativistic $SU(6)_{SF}$ spin-flavor symmetry concerned with light constituent quarks. It is defined in the frame of the relevant hadrons all being at rest, and is called the static $U(12)$ symmetry, $U(12)_{\text{stat}}$, which includes $SU(6)_{SF}$ as a subgroup. As a matter of fact, such an attempt to generalize the $SU(6)_{SF}$ relativistically has a long history and the many critical arguments (for example, No-Go

theorem[22]) against it had been appeared. In this appendix, we shall give a somewhat compact review on the $\tilde{U}(12)$ -classification scheme, and also explain its essential points to overcome the critical points above mentioned.

1. Covariant bi-spinor WF, its charge-conjugation and chiral transformation

In order to represent the $U(12)_{\text{stat}}$ symmetry for the $q\bar{q}$ meson system, we must introduce the relativistically covariant wave function(WF),

$$\Phi_A^B(x, y), \quad A = (\alpha, a), \quad B = (\beta, b), \quad (\text{A1})$$

where α and β are Dirac indices, a and b are flavor indices, and x and y are space-time coordinates of quark and antiquark, respectively. As is explained in the text, the WF of this form is introduced from rather general requirements for the composite hadron system that hadrons should have (i) definite mass and (ii) spin, (iii) definite Lorentz transformation property, and (iv) definite quark-composite structures. We have imaged as a guide the field theoretical expression for WF as

$$\Phi_{M,A}^B(x, y) \sim \langle 0 | \psi_A(x) \bar{\psi}^B(y) | M \rangle + \langle M^c | \psi_A(x) \bar{\psi}^B(y) | 0 \rangle, \quad (\text{A2})$$

where $\psi_A(\bar{\psi}^B)$ denotes the quark field(its Pauli-conjugate), and $|M\rangle(|M^c\rangle)$ denotes the relevant composite meson(its charge-conjugate) state. The WF of charge-

TABLE VII: Ratios of the predicted radiative and pionic decay widths compared with experiments.

Ratio	Experiment	$i\gamma_5\gamma_\mu$	$\gamma_5\sigma_{\mu\nu}$	BEH[9]	God[53]	CFF[54]
$\frac{\Gamma(D_{s0}^X \rightarrow D_s^* \gamma)}{\Gamma(D_{s0}^X \rightarrow D_s \pi^0)}$	$0.29 \pm 0.26[57]$	$0.051^{+0.040}_{-0.016}$	$0.14^{+0.10}_{-0.05}$	0.08	0.2	0.1
$\frac{\Gamma(D_{s1}^X \rightarrow D_s \gamma)}{\Gamma(D_{s1}^X \rightarrow D_s^* \pi^0)}$	$0.44 \pm 0.09[1]$	$0.24^{+0.19}_{-0.07}$	$0.64^{+0.52}_{-0.19}$	0.24	0.6	0.5
$\frac{\Gamma(D_{s1}^X \rightarrow D_s^* \gamma)}{\Gamma(D_{s1}^X \rightarrow D_s^* \pi^0)}$	$0.15 \pm 0.11[57]$	$0.15^{+0.12}_{-0.05}$	$0.40^{+0.33}_{-0.12}$	0.2	0.6	0.2
$\frac{\Gamma(D_{s1}^X \rightarrow D_s^* \gamma)}{\Gamma(D_{s1}^X \rightarrow D_s \gamma)}$	$0.40 \pm 0.28[57]$	0.63		0.9	0.9	0.4
$\frac{\Gamma(D_{s1}^X \rightarrow D_{s0}^X \gamma)}{\Gamma(D_{s1}^X \rightarrow D_s^* \pi^0)}$	$< 0.58[60]$	$(0.61^{+0.49}_{-0.18}) \cdot 10^{-3}$	$(1.7^{+1.3}_{-0.6}) \cdot 10^{-3}$	0.13		

conjugate meson system is represented by

$$\Phi_{M^c, B}^A(y, x) \sim \langle 0 | \psi_B(y) \bar{\psi}^A(x) | M^c \rangle + \langle M | \psi_B(y) \bar{\psi}^A(x) | 0 \rangle. \quad (\text{A3})$$

Then, the Pauli-conjugate WF, defined by $\bar{\Phi} \equiv \gamma_4 \Phi^\dagger \gamma_4$, of Eq. (A2) satisfy the relation

$$\Phi_{M^c, B}^A(y, x) = \overline{\Phi_M(x, y)}_B^A. \quad (\text{A4})$$

These relations imply that the total WF $\Phi_A^B(x, y)$ of the composite meson system and its charge conjugate meson system satisfies the self-conjugate relation,

$$\Phi_A^B(x, y) = \overline{\Phi(y, x)}_A^B, \quad (\text{A5})$$

where

$$\Phi_A^B(x, y) \equiv \sum_M \Phi_{M, A}^B(x, y) = \sum_{M^c} \Phi_{M^c, A}^B(x, y). \quad (\text{A6})$$

The positive frequency part of WF Φ_M is denoted as

$$\Psi_{M, A}^{(+B)}(x, y) \sim \langle 0 | \psi_A(x) \bar{\psi}^B(y) | M \rangle, \quad (\text{A7})$$

which is transformed through charge-conjugation into

$$\longrightarrow \Psi_{M^c, B}^{(+A)}(y, x) \sim \langle 0 | \psi_B(y) \bar{\psi}^A(x) | M^c \rangle, \quad (\text{A8})$$

where $|M^c\rangle = \mathcal{C}|M\rangle$ and \mathcal{C} is the charge conjugation operator. By using the relation

$$\begin{aligned} \mathcal{C}^\dagger \psi_B(y) \mathcal{C} &= C_{BB'} {}^t \bar{\psi}^{B'}(y), \quad C = \gamma_4 \gamma_2 \\ \mathcal{C}^\dagger \bar{\psi}^A(x) \mathcal{C} &= -{}^t \psi_{A'}(x) C^{\dagger A' A}, \quad C^\dagger = \gamma_2 \gamma_4, \end{aligned} \quad (\text{A9})$$

we obtain the relation

$$\Psi_{M^c, B}^{(+A)}(y, x) = C_{BB'} \left({}^t \Psi_M^{(+)}(x, y) \right)^{B'} {}_{A'} C^{\dagger A' A}, \quad (\text{A10})$$

where we use the anti-commutation relation of $\psi_{A'}(x)$ and $\bar{\psi}^{B'}(y)$.

We derive the similar relation to Eq. (A10) between the negative frequency parts, $\Psi_M^{(-)}$ and $\Psi_{M^c}^{(-)}$, of WF Φ_M .

Similarly, the chiral γ_5 -transformation is given by using the operator $\chi(\beta)$ as

$$|M\rangle \longrightarrow |M^{\chi(\beta)}\rangle = \chi(\beta) |M\rangle. \quad (\text{A11})$$

The $\chi(\beta)$ is defined by

$$\chi(\beta)^\dagger \psi_A(x) \chi(\beta) = U(\beta) {}_A^{A'} \psi_{A'}(x), \quad U(\beta) = e^{i \frac{\beta^j \lambda^j}{2} \gamma_5}, \quad (\text{A12})$$

where λ^j are flavor $U(3)_F$ matrices and β^j are the trans-

formation parameters. The WF Φ_A^B is transformed as

$$\begin{aligned} \Phi_A^B(x, y) &= \sum_M \Phi_{M, A}^B(x, y) = \sum_M (\langle 0 | \psi_A(x) \bar{\psi}^B(y) | M \rangle + \langle M^c | \psi_A(x) \bar{\psi}^B(y) | 0 \rangle) \\ &\longrightarrow \sum_M (\langle 0 | \psi_A(x) \bar{\psi}^B(y) | M^{\chi(\beta)} \rangle + \langle M^{\chi(\beta), c} | \psi_A(x) \bar{\psi}^B(y) | 0 \rangle) \\ &= U(\beta) {}_A^{A'} \Phi_{A'}^{B'}(x, y) U(\beta) {}_{B'}^B. \end{aligned} \quad (\text{A13})$$

2. Expansion of bi-spinor WF by the complete set

The Fourier amplitude of $\Phi_A^B(x, y)$ is denoted as $\Phi_A^B(p_1, p_2)$ or $\Phi_A^B(P; q)$ where $p_1(p_2)$ is the momentum of quark(antiquark), and $P_\mu(q_\mu)$ is the CM(internal)

momentum. For the light quark $L\bar{L}$ meson system it is expanded by the complete set of bi-spinor WF on $\tilde{U}(4)$ space $\{ \Gamma_i \}$ (and by the complete set of q_μ -tensors on $O(3, 1)_L$ space) as

$$\begin{aligned} \Phi_A^B(P; q) \sim & \sum_i \phi_i(P)_a {}^b \Gamma_{i,\alpha}{}^\beta f_S(P; q) + \sum_i \phi_{i\mu}(P)_a {}^b \Gamma_{i,\alpha}{}^\beta q_\mu f_P(P; q) \\ & + \sum_i \phi_{i\mu\nu}(P)_a {}^b \Gamma_{i,\alpha}{}^\beta q_\mu q_\nu f_D(P; q) + \dots \end{aligned} \quad (\text{A14})$$

where $\phi_i(P)$'s represent the positive and/or negative frequency Fourier-amplitudes of the respective local meson WF $\phi_i(X)$'s. The first term without an explicit factor q_μ describes the S -wave states, while the second(third) term with a factor $q_\mu(q_\mu q_\nu)$ corresponds to P -wave (D -wave or

radially excited S -wave) states. The summation on the indices i means that on all the 16 component of Dirac γ matrices. The explicit forms of Γ_i and J^{PC} quantum numbers of their corresponding $\phi_i(P)$ are

$$\begin{array}{cccccccc} \Gamma_i & : & 1 & i\gamma_5 & i\tilde{\gamma}_\mu & i\gamma_5\tilde{\gamma}_\mu & -v \cdot \gamma & -\gamma_5 v \cdot \gamma & -i\sigma_{\mu\nu}v_\nu & \gamma_5\sigma_{\mu\nu}v_\nu \\ J^{PC} & : & 0^{++} & 0^{-+} & 1^{--} & 1^{++} & 0^{+-} & 0^{-+} & 1^{--} & 1^{+-} \end{array}, \quad (\text{A15})$$

where $v_\mu = P_\mu/M$ and $i\tilde{\gamma}_\mu = \tilde{\delta}_{\mu\nu}i\gamma_\nu$ ($\tilde{\delta}_{\mu\nu} = \delta_{\mu\nu} + v_\mu v_\nu$), satisfying $v_\mu \tilde{\gamma}_\mu = 0$. For the $HL(Q\bar{q})$ meson system, the expansion of WF is made in the text.

Here it may be instructive to note that the covariant expansion Eq. (15) in the text, leading to Eq. (A14),

applied to WF of the type Eq. (A2) is rather general and is also valid to the other type WF such as the BS amplitude

$$\Phi_A^B(x_1, x_2) \sim \langle 0 | T \psi_A(x_1) \bar{\psi}^B(x_2) | M \rangle \quad (\text{A16})$$

and the gauge-invariant amplitude adopted in Ref.[61],

$$\chi_{\alpha\beta}(x_1, x_2) = \langle 0 | \exp[ig \int_{x_1}^{x_2} A(x) \cdot dx] q_\alpha(x_1) q_\beta^\dagger(x_2) | M \rangle. \quad (\text{A17})$$

3. $U(12)_{\text{stat}}$ symmetry and its representation

The chiral γ_5 -transformation of quark field (A12) induces the chiral transformation of meson WF in

Eq. (A13), which changes the members of $\Gamma_i = i\gamma_5$ and 1 to each other. Thus, the term of pseudoscalar π spinor WF with $\Gamma_i = i\gamma_5$ in Eq. (A15) is transformed into the one of the scalar σ WF with $\Gamma_i = 1$ as

$$\phi_{P_s}(P)_a{}^b (i\gamma_5)_\alpha{}^\beta f_S(P; q) \longrightarrow \phi_S(P)_a{}^b (1)_\alpha{}^\beta f_S(P; q) , \quad (\text{A18})$$

both of which are the S -wave states, and the former(latter) totally represents the pseudoscalar(scalar)

mesons. There exists another scalar nonet from the 3P_0 state, of which WF is given by

$$\Phi_A{}^B(P; q)[^3P_0] \sim f_{0a}{}^b(P) \tilde{\delta}_{\mu\nu} (i\tilde{\gamma}_\mu)_\alpha{}^\beta q_\nu f_P(P; q) . \quad (\text{A19})$$

This WF is contained in the P -wave term, proportional to $\phi_{i\mu}(P)$ in Eq. (A14). Thus, the above two scalar nonets naturally appear in the expansion of our general WF. The one corresponds to the $^3P_0(f_0(1370))$ nonet which is the Pauli state, appearing also in NRQM. The other is the S -wave state corresponding to σ nonet. The σ nonet is degenerate to the π nonet in the ideal case with chiral symmetric phase and they form a linear representation of chiral symmetry. Here it is notable that actually the observed masses of the members of σ nonet

are closer to the π nonet and lower than the $f_0(1370)$ nonet.

Then we notice an interesting possibility that a larger group than $SU(6)_{SF}$ including chiral symmetry is realized in hadron spectroscopy. It is a symmetry $U(12)_{\text{stat}} \supset U(4)_{DS} \times U(3)_F$ combining $U(4)_{DS}$ (which is defined in the rest frame of the relevant hadrons) for Dirac spinor indices and $U(3)_F$ for light flavors, to be called static $U(12)$ symmetry, $U(12)_{\text{stat}}$. Its generators are defined by

$$\begin{aligned} \psi_A &= \psi_{\alpha,a} \rightarrow \psi'_{\alpha,a} = \psi_{\alpha,a} + \delta\psi_{\alpha,a}; \\ \delta\psi_{\alpha,a} &= i(\epsilon^j + \epsilon_5^j \gamma_5 + \epsilon_\mu^j \gamma_\mu + \epsilon_{\mu 5}^j i\gamma_5 \gamma_\mu + \frac{1}{2} \epsilon_{\mu\nu}^j \sigma_{\mu\nu})_\alpha{}^\beta \left(\frac{\lambda^j}{2}\right)_a{}^b \psi_{\beta,b} , \end{aligned} \quad (\text{A20})$$

where ψ_A is the quark field, which belongs to the fundamental **12** representation of $U(12)_{\text{stat}}$. Similarly the antiquark field $\psi^{\dagger B}$ belongs to its conjugate **12*** representation. Here all the **144** infinitesimal parameters ϵ 's are real, and $U(12)_{\text{stat}}$ is a unitary symmetry, where $\psi^\dagger \psi$ is invariant.

As is discussed in the text, it is promising both phenomenologically and theoretically that, *concerning the light quark(antiquark) indices $A(B)$ of WF $\Phi_A{}^B$, all the components of **12(12*)** are treated as physical degrees of freedom. Accordingly the composite hadrons, including light quarks or antiquarks, are considered to be classified with the representation of $U(12)_{\text{stat}}$.*

The light-quark $q\bar{q}$ mesons in the ground S -wave state are classified as **12** \times **12*** = **144** in $U(12)_{\text{stat}}$. Their quantum numbers are given in Eq. (A15), which includes the 0^{-+} and 1^{--} nonets (mixtures of Pauli- and chiral states) forming **6** \times **6*** = **36** in $SU(6)_{SF}$ as well as the 0^{++} σ -nonet (chiral states). All of them are degenerate in the ideal case of $U(12)_{\text{stat}}$ symmetry.

The heavy-light $Q\bar{q}$ -meson system is classified following **6** of $SU(6)_{SF}$ for Q and **12*** of $U(12)_{\text{stat}}$ for \bar{q} . The ground state multiplet of $c\bar{q}$ system includes the 0^+ and 1^+ triplets, which are chiral states appearing newly in $U(12)_{\text{stat}}$, as well as the 0^- and 1^- triplets, which are Pauli-states, also appearing in NRQM.

The light quark qqq baryon system in the S -wave states is classified as **(12** \times **12** \times **12)**_S = **364**, which include baryon and antibaryon. The **182** of baryons is decomposed into **182** = **56** + **70** + **56'**, which includes the conventional **(6** \times **6** \times **6)**_S = **56** in $SU(6)_{SF}$. Additional **70(56')** has negative(positive) parity. The positive parity $N(1440)$, $\Delta(1600)$ and $\Sigma(1660)$ are the candidates of the **56'**, which are the S -wave states and expected to have smaller masses than the ordinary P -wave baryons. Thus, the existence of chiral **56'** in addition to **56** naturally explains the doublings of positive parity states, experimentally confirmed. The negative parity baryons in **70** decay to the **56** baryons and the π -meson octet in S -wave. The overlapping of the WFs is expected to become much larger than those of the decays of excited baryons, since both of the initial and final baryons in the

relevant decays are in ground states. Thus, the baryons in **70** have generally very wide widths and are expected to be observed only as backgrounds, except for the cases of the problematic $\Lambda(1405)$.

Here we should add a following remark.

The $U(12)_{\text{stat}}$ symmetry transformation mixes the flavor and different spin components, as the $SU(6)_{SF}$ does. In order to define this type of symmetry consistently, avoiding an application of No-Go theorem[22], we must specify the frame of relevant hadron (see also the original approach on this line[23]). The $U(12)_{\text{stat}}$ transformation is defined in the frame where all hadrons are at rest. This is the meaning of *static* symmetry. Accordingly the $U(12)_{\text{stat}}$ does *not* include the space-time symmetry for Lorentz boost as a subgroup. In the original $\tilde{U}(12)$ theory all of the homogeneous Lorentz group is included as a subgroup. The $\sigma_{\mu\nu}$ defined in Eq. (A20) are interpreted as its generators only when the ϵ_{4i} are taken to be pure-imaginary, while they are real in $U(12)_{\text{stat}}$. On the other hand the chiral $SU(3)_L \times SU(3)_R$ symmetry is included as a subgroup of $U(12)_{\text{stat}}$. Its linear representation is realized in the $U(12)_{\text{stat}}$ multiplet.

4. $\tilde{U}(12)$ -classification scheme and Lorentz covariance

Our classification scheme of hadrons is based on $U(12)_{\text{stat}}$ symmetry, however, we call it $\tilde{U}(12)$ -

classification scheme. The $\tilde{U}(12)$ transformation is defined by the same equation as Eq. (A20), if we take the transformation parameters $\epsilon_5^j, \epsilon_4^j, \epsilon_{45}^j$ and ϵ_{4i}^j to be pure imaginary, while the other parameters are real. The Lorentz boost is included as a subgroup of $\tilde{U}(12)$ since for the boost the $\epsilon_{\mu\nu}^{j \neq 0} = 0$ and $\epsilon_{4i}^{j=0} (\neq 0)$ is pure imaginary.

The reasons for using the term $\tilde{U}(12)$ in our scheme are as follows:

- i) The representation space of $U(12)_{\text{stat}}$ is the same as the one of $\tilde{U}(12)$ at the rest frame of hadrons. Historically the term, **144(364)** applied for mesons(baryons), was used in the framework of $\tilde{U}(12)$ symmetry[26].
- ii) It is impossible to define $\tilde{U}(12)$ as exact mathematical group generalizing $SU(6)_{SF}$ relativistically according to No-Go theorem[22]. Nevertheless, we can extend the $U(12)_{\text{stat}}$ representation space, which is defined at zero velocity of hadrons, to the space with any velocity covariantly by using the Lorentz booster in $\tilde{U}(12)$.

This can be done through the following procedures. The spinor indices for the WF $\Phi_{A=(\alpha,a)}^{B=(\beta,b)}(\mathbf{v} = \mathbf{0})$ (which forms the $U(12)_{\text{stat}}$ representation space at $\mathbf{v} = \mathbf{0}$) are expanded by the spinors of free-quark type with zero velocity:

$$\begin{aligned}
 & \text{for index } \alpha & \text{for index } \beta \\
 u_{+,s}(\mathbf{0}) &= \begin{pmatrix} \chi^{(s)} \\ 0 \end{pmatrix}, & \bar{v}_{+,\bar{s}}(\mathbf{0}) &= (0, -\chi^{(\bar{s})'}), & \chi^{(\bar{s})'} &= -i\sigma_2 \chi^{(s)*}. \\
 u_{-,s}(\mathbf{0}) &= \begin{pmatrix} 0 \\ \chi^{(s)} \end{pmatrix}, & \bar{v}_{-,\bar{s}}(\mathbf{0}) &= (\chi^{(\bar{s})'}, 0).
 \end{aligned} \tag{A21}$$

Here we should note that $u_{-}(\bar{v}_{-})$ as well as $u_{+}(\bar{v}_{+})$ are required for expansion-bases of Dirac index $\alpha(\beta)$. These

spinors are boosted by free-quark generators $\sigma_{\mu\nu}$ in $\tilde{U}(12)$ into those with non-zero velocity \mathbf{v} as

$$\begin{aligned}
 u_{+,s}(\mathbf{v}) &= \begin{pmatrix} ch\theta \chi^{(s)} \\ sh\theta \mathbf{n} \cdot \sigma \chi^{(s)} \end{pmatrix}, & \bar{v}_{+,\bar{s}}(\mathbf{v}) &= (sh\theta \chi^{(\bar{s})'} \mathbf{n} \cdot \sigma, -ch\theta \chi^{(\bar{s})'}), \\
 u_{-,s}(\mathbf{v}) &= \begin{pmatrix} sh\theta \mathbf{n} \cdot \sigma \chi^{(s)} \\ ch\theta \chi^{(s)} \end{pmatrix}, & \bar{v}_{-,\bar{s}}(\mathbf{v}) &= (ch\theta \chi^{(\bar{s})'}, -sh\theta \chi^{(\bar{s})'} \mathbf{n} \cdot \sigma). \\
 ch\theta &= \sqrt{\frac{E+m}{2m}}, & sh\theta &= \sqrt{\frac{E-m}{2m}}.
 \end{aligned} \tag{A22}$$

By using these spinors, the $U(12)_{\text{stat}}$ representation space

is extended to any velocity, and the Lorentz covariance is

guaranteed for the WF of composite hadron system. This extended scheme is called $\tilde{U}(12)$ classification scheme.

For example, $\Phi(\mathbf{v} = \mathbf{0}) = -i\gamma_5\gamma_4$ corresponds to extra

pseudoscalar state (besides pion state with WF $i\gamma_5$) with $J^{PC} = 0^{-+}$, which is expanded by $u(\mathbf{0})$ and $\bar{v}(\mathbf{0})$ as

$$\Phi(\mathbf{v} = \mathbf{0}) = -i\gamma_5\gamma_4 = \sum_{s\bar{s}} \frac{c_{s\bar{s}}}{i} (u_{+s}(\mathbf{0})\bar{v}_{+\bar{s}}(\mathbf{0}) + u_{-s}(\mathbf{0})\bar{v}_{-\bar{s}}(\mathbf{0})) \quad c_{s\bar{s}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (\text{A23})$$

This WF is boosted to $\Phi(\mathbf{v})$ with the non-zero velocity \mathbf{v} as

$$\Phi(\mathbf{v}) = -\gamma_5 v \cdot \gamma = \sum_{s\bar{s}} \frac{c_{s\bar{s}}}{i} (u_{+s}(\mathbf{v})\bar{v}_{+\bar{s}}(\mathbf{v}) + u_{-s}(\mathbf{v})\bar{v}_{-\bar{s}}(\mathbf{v})), \quad (\text{A24})$$

where the final form $-\gamma_5 v \cdot \gamma$ is also obtained easily from the Lorentz transformation property of γ -matrices. On

the other hand the scalar σ WF with $J^{PC} = 0^{++}$ is common in any velocity frame as

$$\Phi(\mathbf{0}) = \Phi(\mathbf{v}) = 1 = -\sum_{s\bar{s}} c_{s\bar{s}} (u_{+s}(\mathbf{v})\bar{v}_{-\bar{s}}(\mathbf{v}) - u_{-s}(\mathbf{v})\bar{v}_{+\bar{s}}(\mathbf{v})). \quad (\text{A25})$$

5. Urciton spinors and ρ -spin

The spinor WF Eq. (A22) are called urciton spinors. The name, urciton(ur-exciton), is used historically in the exciton quark model proposed[21] by one of the authors 35 years ago, for the purpose of treating multi-quark hadrons systematically and covariantly. In the urciton scheme, each index of the spinor WF is boosted by the same velocity as the relevant hadron.

We should note that in the $\tilde{U}(12)$ -classification scheme the urciton spinors are purely formal objects to be introduced as expansion-bases of hadron WF Φ_A^B . The $u_{+s}(\bar{v}_{+\bar{s}})$ has its correspondents in NRQM or in free-quark field theory, while there is no such correspondence for $u_{-s}(\bar{v}_{-\bar{s}})$. As is explained in section II, conventionally the spinors $u_{-}(v_{-})$ for quarks(antiquarks) are identified with the spinors $v_{+}(u_{+})$ for antiquarks(quarks). This is based upon the hole-theory on the free quark field theory. Correspondingly, in NRQM only the NR two-component Pauli-spinors $\chi^{(s)}(\chi^{(\bar{s})'})$ for quarks(antiquarks), which becomes equivalent to the upper(lower) two-components of four component boosted-Pauli spinors $u_{+}(v_{+})$ in the static limit, are applied. However, this picture on hole theory and the identification of $u_{-} = v_{+}(v_{-} = u_{+})$ is only applicable to the free quarks (or to whole free

hadrons), and unable separately to the indices of confined constituent quarks, coexisting with the other quarks.

In the original urciton scheme, only u_{+} and \bar{v}_{+} are considered as representing physical degrees of freedom. However, the equations (A24) and (A25) suggest the $u_{-}(\bar{v}_{-})$ is also realized as the physical degrees of freedom in composite hadron systems. This freedom for light urciton-quark may be dynamically generated, although it seems a very difficult problem that what is the explicit field theoretical representation of u_{-} and v_{-} .

This new $SU(2)$ freedom, describing u_{+} and u_{-} (v_{+} and v_{-}), is called ρ -spin freedom, while the ordinary $SU(2)$ spin is called σ -spin. These names comes from the $\rho \times \sigma$ decomposition of Dirac γ -matrices. The $u_{+}(\bar{v}_{+})$ have positive $\rho_3(\bar{\rho}_3)$, while $u_{-}(\bar{v}_{-})$ have negative $\rho_3(\bar{\rho}_3)$, where $\bar{\rho}_3 = -\rho_3^t$. The chiral γ_5 -transformation for Φ_A^B is interpreted as the ρ_1 transformation for urciton-spinor space, since $-\gamma_5 u_{\pm} = u_{\mp}$ and $\bar{v}_{\pm} \gamma_5 = \bar{v}_{\mp}$. The states including urciton spinors with negative $\rho_3(\bar{\rho}_3)$ components are called chiral states, while the states including the components with only positive $\rho_3(\bar{\rho}_3)$ are called Pauli states. The Pauli states and chiral states are transformed into each other through chiral transformation, and the linear representation is realized within the members of a single $U(12)_{\text{stat}}$ multiplet.

6. \mathcal{S} matrix unitarity in $\tilde{U}(12)$ scheme

As is explained in the previous subsections, our $\tilde{U}(12)$ group is not defined on physical Hilbert space directly, but is defined on the urciton space. In this connection we should like to note that our $U(12)_{\text{stat.}}$ symmetry seems to be a requirement on the “generalized M function” [27] proposed for the 144-fold way out from the difficulty of $\tilde{U}(12)$. Because of this way our scheme including $SU(6)_{SF}$ becomes consistent with Lorentz covariance. In Ref.[27], the \mathcal{S} matrix is represented by using the clasical free-quark type spinors as

$$\begin{aligned} & \langle \mathbf{p}_1 n_1 \sigma_1, \dots | \mathcal{S} | \mathbf{p}_2 n_2 \sigma_2, \dots \rangle \\ &= \sum_{N_1, N_2} u_{N_1}^* (\mathbf{p}_1 n_1 \sigma_1) \cdots \mathcal{M}_{N_1 \dots, N_2 \dots} (\mathbf{p}_1 \cdots, \mathbf{p}_2 \cdots) \\ & \quad \cdot u_{N_2} (\mathbf{p}_2 n_2 \sigma_2) \cdots, \end{aligned} \quad (\text{A26})$$

where $u_N(\mathbf{p}n\sigma)$ is free Dirac spinor with momentum \mathbf{p} , flavor n and spin $j_z = \sigma$. $N = (\alpha, n')$ is (spinor, flavor) index. There the $\tilde{U}(12)$ transformation is defined as acting on indices $N_1 \cdots$ and $N_2 \cdots$ of the \mathcal{M} -function. The $u_N(\mathbf{p}n\sigma)$ spinors are supposed to be “not WF in the sense of representatives of a state in physical Hilbert space. They are purely formal objects, whose sole purpose in physics is to allow us to define free fields or \mathcal{M} functions[27].” The $u_N(\mathbf{p}n\sigma)$ are physically equivalent to our urciton spinors. Because of this treatment of $\tilde{U}(12)$ symmetry, the resulting \mathcal{S} matrix is shown to be consistent with the Lorentz covariance.

However, in Ref.[27], only u_+ and v_+ (in our terminol-

ogy) are taken as $u_N(\mathbf{p}n\sigma)$ spinors for quark and anti-quark, respectively, and in this prescription the problem of \mathcal{S} -matrix unitarity in the original $U(12)$ seems to remain still unsolved. The requirement of unitarity leads to the non-linear equation for \mathcal{M} -matrix as

$$Im\mathcal{M} = \mathcal{M}\Sigma\mathcal{M}^\dagger, \quad (\text{A27})$$

where the Σ is given, by using $u_N(\mathbf{p}n\sigma)$, symbolically as

$$\begin{aligned} \Sigma_{NN'} &= \sum_{n\sigma} u_N(\mathbf{p}n\sigma) u_{N'}(\mathbf{p}n\sigma)^\dagger \\ &= \sum_s u_{+s, \alpha}(\mathbf{p}) u_{+s, \alpha'}(\mathbf{p})^\dagger \delta_{nn'} \\ &= \left(\frac{1 - iv \cdot \gamma}{2} \gamma_4 \right)_{\alpha\alpha'} \delta_{nn'}. \end{aligned} \quad (\text{A28})$$

This Σ is apparently non-invariant in $\tilde{U}(12)$ transformation, and thus, $\tilde{U}(12)$ violates the unitarity. In our scheme, the above Σ is extended to

$$\begin{aligned} \Sigma'_{NN'} &= \sum_s (u_{+s}(\mathbf{p}) u_{+s}(\mathbf{p})^\dagger + u_{-s}(\mathbf{p}) u_{-s}(\mathbf{p})^\dagger)_{\alpha\alpha'} \delta_{nn'} \\ &= (-iv \cdot \gamma \gamma_4)_{\alpha\alpha'} \delta_{nn'}. \end{aligned} \quad (\text{A29})$$

This Σ' is also not invariant under $\tilde{U}(12)$. However it reduces, in the static limit $v_\mu \rightarrow (\mathbf{0}, i)_\mu$, to $(\gamma_4 \gamma_4)_{\alpha\alpha'} \delta_{nn'} = 1_{\alpha\alpha'} \delta_{nn'} = \delta_{NN'}$, and is invariant under $U(12)_{\text{stat.}}$ transformation. Thus, our $U(12)_{\text{stat}}$ is consistent with the \mathcal{S} matrix unitarity.

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